

Zadatak 29. Za dane funkcije $f(x) = \log_{\frac{1}{2}} \frac{x}{x+1}$ i
 $g(x) = 2^{|x+1|}$ riješi nejednadžbu $(f \circ g)(x) \geq x$.

$$f(x) = \log_{\frac{1}{2}} \frac{x}{x+1}, \quad g(x) = 2^{|x+1|}$$

$$(f \circ g)(x) = \log_{\frac{1}{2}} \frac{2^{|x+1|}}{2^{|x+1|} + 1} = -\log_2 \frac{2^{|x+1|}}{2^{|x+1|} + 1} = \log_2 \frac{2^{|x+1|} + 1}{2^{|x+1|}}$$

$$\log_2 \frac{2^{|x+1|} + 1}{2^{|x+1|}} \geq x = \log_2 2^x \implies \frac{2^{|x+1|} + 1}{2^{|x+1|}} \geq 2^x$$

$$2^{|x+1|} + 1 \geq 2^{x+|x+1|}$$

$$\frac{x}{x+1} > 0 \implies x \in [-1, 0]$$

$$(i) \quad x < -1$$

$$2^{-x-1} + 1 \geq 2^{x-x-1}$$

$$2^{-x-1} + 1 \geq \frac{1}{2}$$

$$2^{-x-1} \geq -\frac{1}{2}$$

$$\implies x \in (-\infty, -1)$$

$$(ii) \quad x > 0$$

$$2^{x+1} + 1 \geq 2^{2x+1}$$

$$2 \cdot 2^{2x} - 2 \cdot 2^x - 1 \leq 0$$

$$(2^x)_{1.2} = \frac{2 \pm \sqrt{4+8}}{4} = \frac{2 \pm 2\sqrt{3}}{4} = \frac{1 \pm \sqrt{3}}{2}$$

$$0 < 2^x \leq \frac{1+\sqrt{3}}{2}$$

$$\implies 0 < x \leq \log_2 \frac{1+\sqrt{3}}{2}$$

$$\implies x \in (-\infty, -1) \cup \left(0, \log_2 \frac{1+\sqrt{3}}{2} \right].$$