Zadatak 31. Ako je $f(x) = \left(\frac{1}{2}\right)^{|x-1|}$, $g(x) = 2\log_4(2x) - \log_{\sqrt{2}} x$, riješi nejednadžbu $(f \circ g)(x) > x^2$.

Rješenje. $f(x) = \left(\dfrac{1}{2}\right)^{|x-1|}$, $g(x) = 2\log_4(2x) - \log_{\sqrt{2}} x$, $x > 0$

$$(f \circ g)(x) = \left(\frac{1}{2}\right)^{|2\log_4(2x) - \log_{\sqrt{2}} x - 1|}$$

$$\left\{2\log_4(2x) - \log_{\sqrt{2}} x - 1 = \log_2 2 + \log_2 x - 2\log_2 x - 1 = -\log_2 x\right\}$$

$$= \left(\frac{1}{2}\right)^{|-\log_2 x|} = 2^{-|-\log_2 x|}$$

(i) $x \in \langle 0, 2\rangle$,

$$2^{-|-\log_2 x|} > x^2 \implies 2^{-\log_2 x} > x^2 \implies \frac{1}{x} > x^2 \implies x^2 - \frac{1}{x} < 0$$

$$\implies \frac{x^3 - 1}{x} < 0 \implies \frac{(x-1)(x^2+x+1)}{x} < 0 \implies x \in \langle 0, 1\rangle$$

(ii) $x \geqslant 2$, $2^{\log_2 x} > x^2 \implies x > x^2 \implies x^2 - x < 0 \implies x(x-1) < 0 \implies x \in \emptyset$

$$\implies x \in \langle 0, 1\rangle.$$