

Zadatak 32. Riješi nejednadžbu $(f \circ g)(x) < 2$ ako je

$$f(x) = \log_x \frac{1}{3} - \log_{\sqrt{3}} x, \quad g(x) = 3^{1-|x|}.$$

Rješenje. $f(x) = \log_x \frac{1}{3} - \log_{\sqrt{3}} x, \quad g(x) = 3^{1-|x|}, \quad x > 0$

$$\begin{aligned} (f \circ g)(x) &= \log_{3^{1-|x|}} \frac{1}{3} - \log_{\sqrt{3}} 3^{1-|x|} = -\log_{3^{1-|x|}} 3 - 2 \log_3 3^{1-|x|} \\ &= -\frac{1}{\log_3 3^{1-|x|}} - 2(1-|x|) = -\frac{1}{1-|x|} - 2(1-|x|) \\ &= \frac{1}{|x|-1} + 2(|x|-1) \end{aligned}$$

$$\frac{1}{|x|-1} + 2(|x|-1) < 2 \implies \frac{1 + 2(|x|-1)^2 - 2(|x|-1)}{|x|-1} < 0$$

$$\implies \frac{2(|x|^2 - 2|x| + 1 - |x| + 2) + 1}{|x|-1} < 0 \implies \frac{2|x|^2 - 6|x| + 7}{|x|-1} < 0$$

$>0 \forall x \in \mathbf{R}$

$$\implies |x| - 1 < 0 \implies |x| < 1$$

$$\implies x \in (0, 1).$$