Zadatak 4. Izračunaj sljedeće limese:

1) $\lim\limits_{x \to 2} \dfrac{x+2}{x^2-4}$;

2) $\lim\limits_{x \to 2} \dfrac{x-2}{x^2-4}$;

3) $\lim\limits_{u \to 1} \dfrac{u-1}{u^2+2u-3}$;

4) $\lim\limits_{u \to 1} \dfrac{u-1}{u^2-2u+1}$;

5) $\lim\limits_{x \to \frac{1}{2}} \dfrac{8x^3-1}{6x^2-5x+1}$;

6) $\lim\limits_{x \to 1} \dfrac{x^3+x-2}{x^3-x^2-x+1}$;

7) $\lim\limits_{x \to 1} \left(\dfrac{1}{1-x} - \dfrac{3}{1-x^3} \right)$;

8) $\lim\limits_{u \to -2} \dfrac{u^3+3u^2+2u}{u^2-2u-8}$.

Rješenje. 1) $\lim\limits_{x \to 2} \dfrac{x+2}{x^2-4}$ ne postoji,

2) $\lim\limits_{x \to 2} \dfrac{x-2}{x^2-4} = \lim\limits_{x \to 2} \dfrac{1}{x+2} = \dfrac{1}{4}$,

3) $\lim\limits_{n \to 1} \dfrac{n-1}{n^2+2n-3} = \lim\limits_{n \to 1} \dfrac{n-1}{(n-1)(n+3)} = \dfrac{1}{4}$,

4) $\lim\limits_{n \to 1} \dfrac{n-1}{n^2-2n+1} = \lim\limits_{n \to 1} \dfrac{1}{n-1}$ ne postoji,

5) $\lim\limits_{x \to \frac{1}{2}} \dfrac{8x^3-1}{6x^2-5x+1} = \lim\limits_{x \to \frac{1}{2}} \dfrac{(2x-1)(4x^2+2x+1)}{(2x-1)(3x-1)} = 6$,

6) $\lim\limits_{x \to 1} \dfrac{x^3+x-2}{x^3-x^2-x+1} = \lim\limits_{x \to 1} \dfrac{(x-1)(x^2+x+2)}{(x-1)^2(x+1)} = \lim\limits_{x \to 1} \dfrac{x^2+x+2}{x^2-1}$ ne postoji,

7)

$$\lim\limits_{x \to 1} \left(\dfrac{1}{1-x} - \dfrac{3}{1-x^3} \right) = \lim\limits_{x \to 1} \dfrac{1+x+x^2-3}{1-x^3} = \lim\limits_{x \to 1} \dfrac{x^2+x-2}{1-x^3}$$

$$= \lim\limits_{x \to 1} \dfrac{(x+2)(x-1)}{(1-x)(x^2+x+1)} = \lim\limits_{x \to 1} \left(-\dfrac{x+2}{x^2+x+1} \right) = -1,$$

8) $\lim\limits_{n \to -2} \dfrac{n^3+3n^2+2n}{n^2-2n-8} = \lim\limits_{n \to -2} \dfrac{n(n+1)(n+2)}{(n+2)(n-4)} = -\dfrac{1}{3}$.