Zadatak 5. Izračunaj sljedeće limese:

1) $\lim\limits_{h\to 0}\dfrac{(x+h)^3-x^3}{h}$; **2)** $\lim\limits_{h\to 0}\dfrac{(x+h)^4-x^4}{h}$;

3) $\lim\limits_{h\to 0}\dfrac{(x+h)^n-x^n}{h}$.

Rješenje. **1)**

$$\begin{aligned}\lim_{h\to 0}\frac{(x+h)^3-x^3}{h} &= \lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+3xh^2+h^3-x^3}{h}\\ &= \lim_{h\to 0}(3x^2+3xh+3xh+h^2)=3x^2;\end{aligned}$$

2)

$$\begin{aligned}\lim_{h\to 0}\frac{(x+h)^4-x^4}{h} &= \lim_{h\to 0}\frac{[(x+h)^2-x^2][(x+h)^2+x^2]}{h}\\ &= \lim_{h\to 0}\frac{(h^2+2hx)(2x^2+2hx-h^2)}{h}\\ &= \lim_{h\to 0}(h+2x)(2x^2+2hx-h^2)\\ &= \lim_{h\to 0}(4x^2+4hx^2-2xh^2+2x^2h+2h^2x-h^3)\\ &= \lim_{h\to 0}(4x^2+6hx^2-h^3)=4x^3;\end{aligned}$$

3)

$$\begin{aligned}\lim_{h\to 0}\frac{(x+h)^n-x^n}{h} &= \lim_{h\to 0}\frac{(x+h-x)(x^{n-1}+x^{n-2}(x+h)+\ldots+x(x+h)^{n-2}+(x+h)^{n-1})}{h}\\ &= \lim_{h\to 0}(x^{n-1}+x^{n-2}(x+h)+\ldots+x(x+h)^{n-2}+(x+h)^{n-1})=nx^{n-1}.\end{aligned}$$