

Zadatak 6. Izračunaj sljedeće limese:

$$1) \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x};$$

$$2) \lim_{x \rightarrow 5} \frac{\sqrt{x} - \sqrt{5}}{x - 5};$$

$$3) \lim_{x \rightarrow 0} \frac{\sqrt[5]{(1+x)^3} - 1}{x};$$

$$4) \lim_{x \rightarrow -1} \frac{\sqrt{4+x+x^2} - 2}{x+1};$$

$$5) \lim_{x \rightarrow 0} \frac{x}{\sqrt[3]{1+x} - 1};$$

$$6) \lim_{x \rightarrow 0} \frac{x}{1 - \sqrt{1-x}};$$

$$7) \lim_{x \rightarrow 1} \frac{\sqrt{x} + \sqrt{x-1} - 1}{\sqrt{x^2-1}};$$

$$8) \lim_{x \rightarrow 1} \frac{x^2 - \sqrt{x}}{\sqrt{x}-1};$$

$$9) \lim_{x \rightarrow 8} \frac{\sqrt{x+1} - 3}{x-8};$$

$$10) \lim_{x \rightarrow 0} \frac{\sqrt{x^2+4} - 2}{\sqrt{x^2+9} - 3}.$$

Rješenje.

1)

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} &= \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} \cdot \frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2} = \lim_{x \rightarrow 0} \frac{x+4-4}{x(\sqrt{x+4} + 2)} \\ &= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+4} + 2} = \frac{1}{4}; \end{aligned}$$

$$2) \lim_{x \rightarrow 5} \frac{\sqrt{x} - \sqrt{5}}{x - 5} = \lim_{x \rightarrow 5} \frac{\sqrt{x} - \sqrt{5}}{(\sqrt{x} - \sqrt{5})(\sqrt{x} + \sqrt{5})} = \lim_{x \rightarrow 5} \frac{1}{\sqrt{x} + \sqrt{5}} = \frac{1}{2\sqrt{5}};$$

3)

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sqrt[5]{(1+x)^3} - 1}{x} &= \left\{ \begin{array}{l} 1+x=t^5 \\ x \rightarrow 0, \quad t \rightarrow 1 \end{array} \right\} = \lim_{t \rightarrow 1} \frac{t^3 - 1}{t^5 - 1} \\ &= \lim_{t \rightarrow 1} \frac{(t-1)(t^2+t+1)}{(t-1)(t^4+t^3+t^2+t+1)} = \frac{3}{5}; \end{aligned}$$

4)

$$\begin{aligned} \lim_{x \rightarrow -1} \frac{\sqrt{4+x+x^2} - 2}{x+1} &= \lim_{x \rightarrow -1} \frac{\sqrt{4+x+x^2} - 2}{x+1} \cdot \frac{\sqrt{4+x+x^2} + 2}{\sqrt{4+x+x^2} + 2} \\ &= \lim_{x \rightarrow -1} \frac{4+x+x^2 - 4}{(x+1)(\sqrt{4+x+x^2} + 2)} = \lim_{x \rightarrow -1} \frac{x(x+1)}{(x+1)(\sqrt{4+x+x^2} + 2)} = -\frac{1}{4}; \end{aligned}$$

5)

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x}{\sqrt[3]{1+x} - 1} &= \lim_{x \rightarrow 0} \frac{x}{\sqrt[3]{1+x} - 1} \cdot \frac{\sqrt[3]{(1+x)^2} + \sqrt[3]{1+x} + 1}{\sqrt[3]{(1+x)^2} + \sqrt[3]{1+x} + 1} \\ &= \lim_{x \rightarrow 0} \frac{x(\sqrt[3]{(1+x)^2} + \sqrt[3]{1+x} + 1)}{1+x-1} = 3 \end{aligned}$$

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$$\lim_{x \rightarrow 0} \frac{x}{\sqrt[3]{1+x} - 1} = \left\{ \begin{array}{l} 1+x = t^3 \\ x \rightarrow 0 \implies t \rightarrow 1 \end{array} \right\} = \lim_{t \rightarrow 1} \frac{t^3 - 1}{t - 1} \\ = \lim_{t \rightarrow 1} \frac{(t-1)(t^2+t+1)}{t-1} = 3;$$

6)

$$\lim_{x \rightarrow 0} \frac{x}{1 - \sqrt{1-x}} = \left\{ \begin{array}{l} 1-x = t^2 \\ x \rightarrow 0, \quad t \rightarrow 1 \end{array} \right\} = \lim_{t \rightarrow 1} \frac{1-t^2}{1-t} \\ = \lim_{t \rightarrow 1} (1+t) = 2;$$

7)

$$\lim_{x \rightarrow 1} \frac{\sqrt{x} + \sqrt{x-1} - 1}{\sqrt{x^2-1}} = \lim_{x \rightarrow 1} \left(\sqrt{\frac{x-1}{x^2-1}} + \frac{\sqrt{x-1}}{\sqrt{x^2-1}} \right) \\ = \lim_{x \rightarrow 1} \left(\sqrt{\frac{1}{x+1}} + \sqrt{\frac{(\sqrt{x}-1)^2}{(\sqrt{x}-1)(\sqrt{x}+1)(x+1)}} \right) \\ = \lim_{x \rightarrow 1} \left(\frac{1}{\sqrt{x+1}} + \sqrt{\frac{\sqrt{x}-1}{(\sqrt{x}+1)(x+1)}} \right) = \frac{1}{\sqrt{2}},$$

8)

$$\lim_{x \rightarrow 1} \frac{x^2 - \sqrt{x}}{\sqrt{x}-1} = \left\{ \begin{array}{l} \sqrt{x} = t \\ x \rightarrow 1, \quad t \rightarrow 1 \end{array} \right\} = \lim_{t \rightarrow 1} \frac{t^4 - t}{t - 1} \\ = \lim_{t \rightarrow 1} \frac{t(t-1)(t^2+t+1)}{t-1} = \lim_{t \rightarrow 1} (t^3 + t^2 + t) = 3;$$

9)

$$\lim_{x \rightarrow 8} \frac{\sqrt{x+1} - 3}{x - 8} = \left\{ \begin{array}{l} x+1 = t^2 \\ x \rightarrow 8, \quad t \rightarrow 3 \end{array} \right\} = \lim_{t \rightarrow 3} \frac{t-3}{t^2 - 9} = \lim_{t \rightarrow 3} \frac{1}{t+3} = \frac{1}{6};$$

10)

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+4} - 2}{\sqrt{x^2+9} - 3} = \frac{\lim_{x \rightarrow 0} \frac{\sqrt{x^2+4} - 2}{1}}{\lim_{x \rightarrow 0} \frac{\sqrt{x^2+9} - 3}{1}} = \frac{\lim_{x \rightarrow 0} \frac{(\sqrt{x^2+4} - 2)(\sqrt{x^2+4} + 2)}{\sqrt{x^2+4} + 2}}{\lim_{x \rightarrow 0} \frac{(\sqrt{x^2+9} - 3)(\sqrt{x^2+9} + 3)}{\sqrt{x^2+9} + 3}} \\ = \frac{\lim_{x \rightarrow 0} \frac{x^2}{\sqrt{x^2+4} + 2}}{\lim_{x \rightarrow 0} \frac{\sqrt{x^2+9} + 3}{\sqrt{x^2+4} + 2}} = \lim_{x \rightarrow 0} \frac{\sqrt{x^2+9} + 3}{\sqrt{x^2+4} + 2} = \frac{6}{4} = \frac{3}{2}.$$