Zadatak 7. Izračunaj sljedeće limese:

1) $\lim\limits_{x\to\infty} \dfrac{3x+2}{5x-1}$;

2) $\lim\limits_{t\to\infty} \dfrac{5-3t-t^2}{5t^2-1}$;

3) $\lim\limits_{x\to\infty} \dfrac{x^2-2x+2}{(x+1)^3-(x-1)^3}$;

4) $\lim\limits_{x\to\infty} \left(\dfrac{x}{3+x} - \dfrac{2x^2}{4+x^2} \right)$;

5) $\lim\limits_{x\to\infty} \dfrac{2+3x-x^2}{3x^2+2x-1}$;

6) $\lim\limits_{x\to\infty} \dfrac{x^4+2x}{1-x^2+3x^4}$;

7) $\lim\limits_{x\to\infty} \left(\dfrac{x}{3+x} - \dfrac{2x^2}{3+x^2} \right)$;

8) $\lim\limits_{x\to\infty} \left(\dfrac{x-x^2}{1-2x^2} - \dfrac{1+3x}{5x+1} \right)$.

Rješenje.

$$P_m(x) = a_m x^m + \ldots$$
$$Q_n(x) = b_n x^n + \ldots$$
$$\lim_{x\to\infty} \frac{P_m(x)}{Q_n(x)} = \left\{ \begin{array}{ll} 0, & m < n \\ \infty, & m > n \\ \dfrac{a_m}{b_m}, & m = n \end{array} \right\}$$

1) $\lim\limits_{x\to\infty} \dfrac{3x+2}{5x-1} = \lim\limits_{x\to\infty} \dfrac{3+\dfrac{2}{x}}{5-\dfrac{1}{x}} = \dfrac{3}{5}$;

2) $\lim\limits_{t\to\infty} \dfrac{5-3t-t^2}{5t^2-1} \; \lim\limits_{t\to\infty} \dfrac{\dfrac{5}{t^2}-\dfrac{3}{t}-1}{5-\dfrac{1}{t^2}} = -\dfrac{1}{5}$;

3)

$$\lim_{x\to\infty} \frac{x^2-2x+2}{(x+1)^3-(x-1)^3} = \lim_{x\to\infty} \frac{x^2-2x+2}{x^3+3x^2+3x+1-x^3+3x^2-3x+1}$$
$$= \lim_{x\to\infty} \frac{x^2-2x+2}{6x^2+2} = \lim_{x\to\infty} \frac{1-\dfrac{2}{x}+\dfrac{2}{x^2}}{6+\dfrac{2}{x^2}} = \frac{1}{6};$$

4) $\lim\limits_{x\to\infty} \left(\dfrac{x}{3+x} - \dfrac{2x^2}{4+x^2} \right) \lim\limits_{x\to\infty} \left(\dfrac{1}{\dfrac{3}{x}+1} - \dfrac{2}{\dfrac{4}{x^2}+1} \right) = 1 - 2 = -1$;

5) $\lim\limits_{x\to\infty} \dfrac{2 + 3x - x^2}{3x^2 + 2x - 1} \lim\limits_{x\to\infty} \dfrac{\frac{2}{x^2} + \frac{3}{x} - 1}{3 + \frac{2}{x} - \frac{1}{x^2}} = -\dfrac{1}{3},$

6) $\lim\limits_{x\to\infty} \dfrac{x^4 + 2x}{1 - x^2 + 3x^4} = \lim\limits_{x\to\infty} \dfrac{1 + \frac{2}{x^3}}{\frac{1}{x^4} - \frac{1}{x^2} + 3} = \dfrac{1}{3},$

7) $\lim\limits_{x\to\infty} \left(\dfrac{x}{3 + x} - \dfrac{2x^2}{3 + x^2} \right) \lim\limits_{x\to\infty} \left(\dfrac{1}{\frac{3}{x} + 1} - \dfrac{2}{\frac{3}{x^2} + 1} \right) = 1 - 2 = -1,$

8) $\lim\limits_{x\to\infty} \left(\dfrac{x - x^2}{1 - 2x^2} - \dfrac{1 + 3x}{5x + 1} \right) = \lim\limits_{x\to\infty} \left(\dfrac{\frac{1}{x} - 1}{\frac{1}{x^2} - 2} - \dfrac{\frac{1}{x} + 3}{5 + \frac{1}{x}} \right) = \dfrac{1}{2} - \dfrac{3}{5} = -\dfrac{1}{10}.$