

**Zadatak 8.** Izračunaj sljedeće limese:

- 1)  $\lim_{x \rightarrow \infty} (\sqrt{x-a} - \sqrt{x})$ ;
- 2)  $\lim_{x \rightarrow \infty} (\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x})$ ;
- 3)  $\lim_{x \rightarrow \infty} (\sqrt[3]{x+1} - \sqrt[3]{x})$ ;
- 4)  $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 - a^2})$ ;
- 5)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + x} + \sqrt{x^2 - 1}}{\sqrt{4x^2 + 3} + 2x}$ ;
- 5)  $\lim_{x \rightarrow \infty} (\sqrt[4]{x+1} - \sqrt[4]{x})$ .

**Rješenje.**

1)

$$\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{x-a} - \sqrt{x}) &= \lim_{x \rightarrow \infty} \frac{(\sqrt{x-a} - \sqrt{x})(\sqrt{x-a} + \sqrt{x})}{\sqrt{x-a} + \sqrt{x}} \\ &= \lim_{x \rightarrow \infty} \frac{-a}{\sqrt{x-a} + \sqrt{x}} = 0; \end{aligned}$$

2)

$$\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}) &= \lim_{x \rightarrow \infty} \frac{(\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x})(\sqrt{x + \sqrt{x + \sqrt{x}}} + \sqrt{x})}{\sqrt{x + \sqrt{x + \sqrt{x}}} + \sqrt{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{x + \sqrt{x}}}{\sqrt{x + \sqrt{x + \sqrt{x}}} + \sqrt{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{1 + \frac{1}{\sqrt{x}}}}{\sqrt{1 + \sqrt{\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{x^3}}}} + 1} = \frac{1}{2}; \end{aligned}$$

$$3) \lim_{x \rightarrow \infty} (\sqrt[3]{x+1} - \sqrt[3]{x}) = \lim_{x \rightarrow \infty} \frac{x+1-x}{\sqrt[3]{(x+1)^2} + \sqrt[3]{x^2+x} + \sqrt[3]{x^2}} = 0;$$

$$4) \lim_{x \rightarrow \infty} (x - \sqrt{x^2 - a^2}) = \lim_{x \rightarrow \infty} \frac{x^2 - x^2 + a^2}{x + \sqrt{x^2 - a^2}} = 0;$$

$$5) \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + x} + \sqrt{x^2 - 1}}{\sqrt{4x^2 + 3} + 2x} = \lim_{x \rightarrow \infty} \frac{\sqrt{1 + \frac{1}{x}} + \sqrt{1 - \frac{1}{x^2}}}{\sqrt{4 + \frac{3}{x^2}} + 2} = \frac{1+1}{2+2} = \frac{2}{4} =$$

$$\frac{1}{2}.$$