

**Zadatak 9.** Izračunaj sljedeće limese:

$$1) \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x};$$

$$2) \lim_{x \rightarrow 0} \frac{\sin x}{\operatorname{tg} 2x};$$

$$3) \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x};$$

$$4) \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2};$$

$$5) \lim_{x \rightarrow 0} \frac{\operatorname{tg} 2x}{\operatorname{tg} x};$$

$$6) \lim_{x \rightarrow 0} \frac{\sin 2x}{\operatorname{tg} x}.$$

*Rješenje.*

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$1) \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x} = \lim_{x \rightarrow 0} \frac{3x \cdot \frac{\sin 3x}{3x}}{5x \cdot \frac{\sin 5x}{5x}} = \frac{3}{5};$$

$$2) \lim_{x \rightarrow 0} \frac{\sin x}{\operatorname{tg} 2x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot x \cdot \frac{\cos 2x}{\sin 2x} = \lim_{x \rightarrow 0} \frac{\cos 2x}{2} \cdot \frac{1}{\frac{\sin 2x}{2x}} = \frac{1}{2};$$

$$3) \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{x^2}{2 \sin^2 \frac{x}{2}} = \lim_{x \rightarrow 0} \frac{1}{2 \frac{\sin^2 \frac{x}{2}}{x^2}} = \lim_{x \rightarrow 0} \frac{1}{2 \cdot \frac{\sin^2 \frac{x}{2}}{(\frac{x}{2})^2 \cdot 4}}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\frac{1}{2} \left( \frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2} = 2;$$

$$4) \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2} = \lim_{x \rightarrow 0} 2 \left( \frac{\sin x}{x} \right)^2 = 2;$$

$$5) \lim_{x \rightarrow 0} \frac{\operatorname{tg} 2x}{\operatorname{tg} x} = \lim_{x \rightarrow 0} \frac{\frac{\sin 2x}{\cos 2x}}{\frac{\sin x}{\cos x}} = \lim_{x \rightarrow 0} \frac{\cos x}{\cos 2x} \cdot 2 \cdot \frac{\sin 2x}{\sin x} = 2;$$

$$6) \lim_{x \rightarrow 0} \frac{\sin 2x}{\operatorname{tg} x} = \lim_{x \rightarrow 0} \cos x \frac{\sin 2x}{\sin x} = \lim_{x \rightarrow 0} 2 \cos x \frac{\sin 2x}{\sin x} = 2.$$