Zadatak 11. Koristeći računalo pokušaj odrediti sljedeće limese:

1) $\lim\limits_{x\to 0}\dfrac{\cos x-1}{x}$;

2) $\lim\limits_{x\to 1^-}\dfrac{\sin\sqrt{1-x}}{\sqrt{1-x^2}}$;

3) $\lim\limits_{x\to 0^+}\dfrac{x-\sqrt{x}}{\sqrt{\sin x}}$.

Rješenje. L'Hospital

1) $\lim\limits_{x\to 0}\dfrac{\cos x-1}{x}=\lim\limits_{x\to 0}\left(-\dfrac{2\sin^2\frac{x}{2}}{x}\right)=\lim\limits_{x\to 0}\dfrac{-2x}{4}\dfrac{\sin^2\frac{x}{2}}{\frac{x^2}{4}}=0$;

2) $\lim\limits_{x\to 1^-}\dfrac{\sin\sqrt{1-x}}{\sqrt{1-x^2}}=\left\{\begin{matrix}1-x=u^2\\ x\to 1^-\\ u\to 0^+\end{matrix}\right\}=\lim\limits_{u\to 0^+}\dfrac{\sin u}{u}\cdot\dfrac{1}{\sqrt{2-u^2}}=\dfrac{1}{\sqrt{2}}$;

3) $\lim\limits_{x\to 0^+}\dfrac{x-\sqrt{x}}{\sqrt{\sin x}}=\lim\limits_{x\to 0^+}\left(\sqrt{\dfrac{x^2}{\sin x}}-\sqrt{\dfrac{x}{\sin x}}\right)=\lim\limits_{x\to 0^+}\left(\sqrt{\dfrac{x}{\frac{\sin x}{x}}}-\sqrt{\dfrac{1}{\frac{\sin x}{x}}}\right)=$
$0-1=-1$.