

**Zadatak 12.** Izračunaj sljedeće limese:

- 1)  $\lim_{x \rightarrow \infty} \left( \frac{x}{1+x} \right)^x$ ;
- 2)  $\lim_{x \rightarrow \infty} \left( 1 + \frac{1}{x} \right)^{\frac{x^2+1}{x}}$ ;
- 3)  $\lim_{x \rightarrow \infty} \left( \frac{x+2}{x+1} \right)^{x+3}$ ;
- 4)  $\lim_{x \rightarrow \infty} \left( \frac{x^2-1}{x^2+1} \right)^{x^2-2}$ ;
- 5)  $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x}$ ;
- 6)  $\lim_{x \rightarrow 0} \frac{\ln(x+2) - \ln 2}{x}$ .

**Rješenje.**

$$\lim_{x \rightarrow \infty} \left( 1 + \frac{1}{x} \right)^x = e$$

$$1) \lim_{x \rightarrow \infty} \left( \frac{x}{1+x} \right)^x = \lim_{x \rightarrow \infty} \left( \frac{x+1}{x} \right)^{-x} = \lim_{x \rightarrow \infty} \left[ \left( 1 + \frac{1}{x} \right)^x \right]^{-1} = e^{-1};$$

$$2) \lim_{x \rightarrow \infty} \left( 1 + \frac{1}{x} \right)^{\frac{x^2+1}{x}} = \lim_{x \rightarrow \infty} \left[ \left( 1 + \frac{1}{x} \right)^x \right]^{\frac{x^2+1}{x^2}} = e^1 = e;$$

$$3) \lim_{x \rightarrow \infty} \left( \frac{x+2}{x+1} \right)^{x+3} = \lim_{x \rightarrow \infty} \left[ \left( 1 + \frac{1}{x+1} \right)^{x+1} \right]^{\frac{x+3}{x+1}} = e^1 = e;$$

$$4) \lim_{x \rightarrow \infty} \left( \frac{x^2-1}{x^2+1} \right)^{x^2-2} = \lim_{x \rightarrow \infty} \left( \frac{x^2+1-2}{x^2+1} \right)^{x^2-2} = \lim_{x \rightarrow \infty} \left[ \left( 1 + \frac{1}{\frac{x^2+1}{-2}} \right)^{\frac{x^2+1}{-2}} \right]^{\frac{(x^2-2)(-2)}{x^2+1}}$$

$$= \lim_{x \rightarrow \infty} \frac{-2x^2+4}{x^2+1} = e^{-2};$$

$$5) \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = \lim_{x \rightarrow 0} \ln(1+x)^{\frac{1}{x}} = \left\{ \begin{array}{l} x = \frac{1}{t} \\ x \rightarrow 0 \\ t \rightarrow \infty \end{array} \right\} = \ln \lim_{t \rightarrow \infty} \left( 1 + \frac{1}{t} \right)^t =$$

$$\ln e = 1;$$

$$6) \lim_{x \rightarrow 0} \frac{\ln(x+2) - \ln 2}{x} = \lim_{x \rightarrow 0} \frac{1}{x} \ln \frac{x+2}{2} = \ln \lim_{x \rightarrow 0} \left( 1 + \frac{1}{\frac{x}{2}} \right)^{\frac{1}{x}} = \left\{ \begin{array}{l} x = \frac{1}{t} \\ x \rightarrow 0 \\ t \rightarrow \infty \end{array} \right\} =$$

$$\ln \lim_{t \rightarrow \infty} \left[ \left( 1 + \frac{1}{2t} \right)^{2t} \right]^{\frac{1}{2}} = \ln e^{\frac{1}{2}} = \frac{1}{2}.$$