Zadatak 6. Za zadane funkcije f i g odredi $f\circ g$ i $g\circ f$ i ispitaj gdje su te funkcije neprekinute:

1) $f(x)=x-1$, $g(x)=\dfrac{1}{x}$;

2) $f(x)=x^3$, $g(x)=\sqrt{x}$;

3) $f(x)=\dfrac{x-1}{x+1}$, $g(x)=\sqrt{x}$;

4) $f(x)=\dfrac{x-1}{x+1}$, $g(x)=x+1$.

Rješenje. **1)** $f(x)=x-1$, $g(x)=\dfrac{1}{x}$,

$$(f\circ g)(x)=\frac{1}{x}-1$$

$$(g\circ f)(x)=\frac{1}{x-1}$$

$$\lim_{x\to 0^-}\left(\frac{1}{x}-1\right)=-\infty,\quad \lim_{x\to 0^+}\left(\frac{1}{x}-1\right)=+\infty \implies \text{prekid u}\quad x=0$$

$$\lim_{x\to 1^-}\frac{1}{x-1}=-\infty,\quad \lim_{x\to 1^+}\frac{1}{x-1}=+\infty \implies \text{prekid u}\quad x=1$$

2) $f(x)=x^3$, $g(x)=\sqrt{x}$,

$$\left.\begin{array}{l}(f\circ g)(x)=\sqrt{x}^3=x^{\frac{3}{2}}\\ (g\circ f)(x)=\sqrt{x^3}=x^{\frac{3}{2}}\end{array}\right\}\quad \text{neprekidna funkcija na}\quad D_{f\circ g}=D_{g\circ f}=\mathbf{R}_0^+$$

3) $f(x)=\dfrac{x-1}{x+1}$, $g(x)=\sqrt{x}$,

$$(f\circ g)(x)=\frac{\sqrt{x}-1}{\sqrt{x}+1},\quad D_{f\circ g}=\mathbf{R},\quad \text{neprekidna}$$

$$(g\circ f)(x)=\sqrt{\frac{x-1}{x+1}},\quad \frac{x-1}{x+1}\geqslant 0 \implies x\in \mathbf{R}/[-1,1\rangle$$

$$\left.\begin{array}{l}\lim\limits_{x\to -1^-}\sqrt{\frac{x-1}{x+1}}=+\infty\\ \lim\limits_{x\to 1^+}\sqrt{\frac{x-1}{x+1}}=0\end{array}\right\} D_{g\circ f}=\langle -\infty,-1\rangle\cup[1,+\infty\rangle\quad \text{neprekidna na}\quad D_{g\circ f}$$

4) $f(x)=\dfrac{x-1}{x+1}$, $g(x)=x+1$,

$$(f\circ g)(x)=\frac{x+1-1}{x+1+1}=\frac{x}{x+2}$$

$$(g\circ f)(x)=\frac{x-1}{x+1}+1=\frac{x-1+x+1}{x+1}=\frac{2x}{x+1}$$

$$\left.\begin{array}{l}\lim\limits_{x\to -2^-}\dfrac{x}{x+2}=+\infty\\ \lim\limits_{x\to -2^+}\dfrac{x}{x+2}=-\infty\end{array}\right\}\quad \text{prekid u}\quad x=-2$$

$$\left.\begin{aligned}\lim_{x\to -1^-}\frac{2x}{x+1}&=+\infty\\ \lim_{x\to -1^+}\frac{2x}{x+1}&=-\infty\end{aligned}\right\}\quad \text{prekid u}\quad x=-1$$