Zadatak 2. Dokaži da za linearnu funkciju $f(x) = ax + b$ te realne brojeve k_1 i k_2 takve da je $k_1 + k_2 \neq 0$ vrijedi:

1) $f(k_1 x_1 + k_2 x_2) = k_1 f(x_1) + k_2 f(x_2)$
$\iff k_1 + k_2 = 1$;

2) $f\left(\dfrac{k_1 x_1 + k_2 x_2}{k_1 + k_2}\right) = \dfrac{k_1 f(x_1) + k_2 f(x_2)}{k_1 + k_2}$.

Rješenje.   **1)** $f(k_1 x_1 + k_2 x_2) = a(k_1 x_1 + k_2 x_2) + b$;

$k_1 f(x_1) + k_2 f(x_2) = ak_1 x_1 + k_1 b + ak_2 x_2 + k_2 b = ak_1 x_1 + ak_2 x_2 + b(k_1 + k_2)$;

$a(k_1 x_1 + k_2 x_2) + b = ak_1 x_1 + ak_2 x_2 + b(k_1 + k_2) \iff b = b(k_1 + k_2)$
$$\iff 1 = k_1 + k_2.$$

2)

$$\begin{aligned}
f\left(\frac{k_1 x_1 + k_2 x_2}{k_1 + k_2}\right) &= a\frac{k_1 x_1 + k_2 x_2}{k_1 + k_2} + b = \frac{ak_1 x_1 + bk_1 + ak_2 x_2 + bk_2}{k_1 + k_2} \\
&= \frac{k_1(ax_1 + b) + k_2(ax_2 + b)}{k_1 + k_2} = \frac{k_1 f(x_1) + k_2 f(x_2)}{k_1 + k_2}.
\end{aligned}$$