Zadatak 3. Dokaži da za polinom drugog stupnja $f(x) = ax^2 + bx + c$ vrijedi:
$f(x+3) - 3f(x+2) + 3f(x+1) - f(x) = 0$ za sve $x \in \mathbf{R}$.

Rješenje.

$$\begin{aligned} & f(x+3) - 3f(x+2) + 3f(x+1) - f(x) \\ &= a(x+3)^2 + b(x+3) + c - 3a(x+2)^2 - 3b(x+2) - 3c + 3a(x+1)^2 + 3b(x+1) + 3c - ax^2 - bx - c \\ &= a[(x+3)^2 - 3(x+2)^2 + 3(x+1)^2 - x^2] + b[x + 3 - 3x - 6 + 3x + 3 - x] \\ &= a[x^2 + 6x + 9 - 3(x^2 + 4x + 4) + 3(x^2 + 2x + 1) - x^2] \\ &= a(x^2 + 6x + 9 - 3x^2 - 12x - 12 + 3x^2 + 6x + 3 - x^2) = 0. \end{aligned}$$