Zadatak 5. Dokaži da za realnu funkciju $f(x) = \log \dfrac{1-x}{1+x}$, $x \in \langle -1, 1 \rangle$ vrijedi $f(x_1) + f(x_2) = f\left(\dfrac{x_1+x_2}{1+x_1x_2}\right)$.

Rješenje. $f(x_1) + f(x_2) = \log \dfrac{1-x_1}{1+x_1} + \log \dfrac{1-x_2}{1+x_2} = \log(1-x_1) - \log(1+x_1) + \log(1-x_2) - \log(1+x_2)$

$$
\begin{aligned}
f\left(\frac{x_1+x_2}{1+x_1x_2}\right) &= \log \frac{1 - \dfrac{x_1+x_2}{1+x_1x_2}}{1 + \dfrac{x_1+x_2}{1+x_1x_2}} = \log \frac{1+x_1x_2-x_1-x_2}{1+x_1x_2+x_1+x_2} \\
&= \log \frac{1-x_1-x_2(1-x_1)}{1+x_1+x_2(1+x_1)} = \log \frac{(1-x_1)(1-x_2)}{(1+x_1)(1+x_2)} \\
&= \log(1-x_1) + \log(1-x_2) - \log(1+x_1) - \log(1+x_2) \\
&= \log \frac{1-x_1}{1+x_1} + \log \frac{1-x_2}{1+x_2} = f(x_1) + f(x_2).
\end{aligned}
$$