Zadatak 13. Funkcije $f_i : \mathbf{R} \setminus \{0, 1\} \rightarrow \mathbf{R}$, $i = 1, 2, \ldots, 6$ zadane su formulama:
$f_1(x) = x$, $f_2(x) = \dfrac{1}{1-x}$, $f_3(x) = \dfrac{x-1}{x}$, $f_4(x) = \dfrac{1}{x}$, $f_5(x) = \dfrac{x}{x-1}$,
$f_6(x) = 1 - x$. Pokaži da je kompozicija bilo kojih dviju od ovih šest funkcija neka od tih funkcija.

Rješenje.

$$(f_1 \circ f_1)(x) = x = f_1(x);$$

$$(f_2 \circ f_1)(x) = \frac{1}{1-x} = f_2(x);$$

$$(f_3 \circ f_1)(x) = \frac{x-1}{x} = f_3(x);$$

$$(f_4 \circ f_1)(x) = \frac{1}{x} = f_4(x);$$

$$(f_5 \circ f_1)(x) = \frac{x}{x-1} = f_5(x);$$

$$(f_6 \circ f_1)(x) = 1 - x = f_6(x);$$

$$(f_1 \circ f_2)(x) = \frac{1}{1-x} = f_2(x);$$

$$(f_2 \circ f_2)(x) = \frac{1}{1 - \frac{1}{1-x}} = \frac{x-1}{x} = f_3(x);$$

$$(f_3 \circ f_2)(x) = \frac{\frac{1}{1-x} - 1}{\frac{1}{1-x}} = x = f_1(x);$$

$$(f_4 \circ f_2)(x) = \frac{1}{\frac{1}{1-x}} = 1 - x = f_6(x);$$

$$(f_5 \circ f_2)(x) = \frac{\frac{1}{1-x}}{\frac{1}{1-x} - 1} = \frac{1}{x} = f_4(x);$$

$$(f_6 \circ f_2)(x) = 1 - \frac{1}{1-x} = \frac{x}{x-1} = f_5(x);$$

$$(f_1 \circ f_3)(x) = \frac{x-1}{x} = f_3(x);$$

$$(f_2 \circ f_3)(x) = \frac{1}{1 - \frac{x-1}{x}} = \frac{x}{x-x+1} = x = f_1(x);$$

$$(f_3 \circ f_3)(x) = \frac{\frac{x-1}{x} - 1}{\frac{x-1}{x}} = \frac{x-1-x}{x-1} = \frac{1}{1-x} = f_2(x);$$

$$(f_4 \circ f_3)(x) = \frac{1}{\frac{x-1}{x}} = \frac{x}{x-1} = f_5(x);$$

$$(f_5 \circ f_3)(x) = \frac{\frac{x-1}{x}}{\frac{x-1}{x} - 1} = 1 - x = f_6(x);$$

$$(f_6 \circ f_3)(x) = 1 - \frac{x-1}{x} = \frac{1}{x} = f_4(x);$$

$$(f_1 \circ f_4)(x) = \frac{1}{x} = f_4(x);$$

$$(f_2 \circ f_4)(x) = \frac{1}{1 - \dfrac{1}{x}} = \frac{x}{x-1} = f_5(x);$$

$$(f_3 \circ f_4)(x) = \frac{\dfrac{1}{x} - 1}{\dfrac{1}{x}} = 1 - x = f_6(x);$$

$$(f_4 \circ f_4)(x) = \frac{1}{\dfrac{1}{x}} = x = f_1(x);$$

$$(f_5 \circ f_4)(x) = \frac{\dfrac{1}{x}}{\dfrac{1}{x} - 1} = \frac{1}{1-x} = f_2(x);$$

$$(f_6 \circ f_4)(x) = 1 - \frac{1}{x} = \frac{x-1}{x} = f_3(x);$$

$$(f_1 \circ f_5)(x) = \frac{x}{x-1} = f_5(x);$$

$$(f_2 \circ f_5)(x) = \frac{1}{1 - \dfrac{x}{x-1}} = \frac{x-1}{x-1-x} = 1 - x = f_6(x);$$

$$(f_3 \circ f_5)(x) = \frac{\dfrac{x}{x-1} - 1}{\dfrac{x}{x-1}} = \frac{x - x + 1}{x} = \frac{1}{x} = f_4(x);$$

$$(f_4 \circ f_5)(x) = \frac{1}{\dfrac{x}{x-1}} = \frac{x-1}{x} = f_3(x);$$

$$(f_5 \circ f_5)(x) = \frac{\dfrac{x}{x-1}}{\dfrac{x}{x-1} - 1} = \frac{x}{x - x + 1} = x = f_1(x);$$

$$(f_6 \circ f_5)(x) = 1 - \frac{x}{x-1} = \frac{x-1-x}{x-1} = \frac{1}{1-x} = f_2(x);$$

$$(f_1 \circ f_6)(x) = 1 - x = f_6(x);$$

$$(f_2 \circ f_6)(x) = \frac{1}{1 - (1-x)} = \frac{1}{x} = f_4(x);$$

$$(f_3 \circ f_6)(x) = \frac{1 - x - 1}{1-x} = \frac{x}{x-1} = f_5(x);$$

$$(f_4 \circ f_6)(x) = \frac{1}{1-x} = f_2(x);$$

$$(f_5 \circ f_6)(x) = \frac{1-x}{1-x-1} = \frac{x-1}{x} = f_3(x);$$

$$(f_6 \circ f_6)(x) = 1 - (1-x) = x = f_1(x).$$