

Zadatak 16. Riješi jednađbu $(f \circ g)(x) = (g \circ f)(x)$ ako je:

1) $f(x) = |x + 1|$, $g(x) = |x| - 2$;

2) $f(x) = x^2 - 2|x|$, $g(x) = x + 1$.

Rješenje.

1)

$$(f \circ g)(x) = ||x| - 2 + 1| = ||x| - 1|;$$

$$(g \circ f)(x) = |x + 1| - 2;$$

$$(f \circ g)(x) = (g \circ f)(x) \implies ||x| - 1| = |x + 1| - 2$$

	-1	0	1
$ x + 1 $	-	+	+
$ x - 1 $	+	-	-
$ x $	-	-	+

$x \in \langle -\infty, -1] \implies$

$$-x - 1 - 2 = |x| - 1,$$

$$-x - 1 - 2 = -x - 1,$$

$$-2 = 0,$$

$$\emptyset$$

$x \in \langle -1, 0) \implies$

$$x + 1 - 2 = -|x| + 1,$$

$$x - 1 = x + 1,$$

$$-1 = 1,$$

$$\emptyset$$

$x \in [0, 1] \implies$

$$x + 1 - 2 = -|x| + 1,$$

$$x - 1 = -x + 1,$$

$$2x = 2$$

$$x = 1$$

$x \in \langle 1, \infty) \implies$

$$x + 1 - 2 = |x| - 1,$$

$$x - 1 = x - 1,$$

$$x \in \langle 1, \infty).$$

Rješenje je $x \in [1, \infty)$.

2)

$$(f \circ g)(x) = (x + 1)^2 - 2|x + 1|;$$

$$(g \circ f)(x) = x^2 - 2|x| + 1;$$

$$(f \circ g)(x) = (g \circ f)(x) \implies (x + 1)^2 - 2|x + 1| = x^2 - 2|x| + 1$$

	-1	0
$(f \circ g)(x)$	$(x+1)^2 + 2(x+1)$	$(x+1)^2 - 2(x+1)$
$(g \circ f)(x)$	$x^2 + 2x + 1$	$x^2 - 2x + 1$

$$\underline{x \in \langle -\infty, -1 \rangle} \implies$$

$$(x+1)^2 + 2(x+1) = x^2 + 2x + 1,$$

$$(x+1)(x+3) = (x+1)^2,$$

$$x+3 = x+1,$$

$$3 = 1,$$

$$\emptyset$$

$$\underline{x \in [-1, 0]} \implies$$

$$(x+1)^2 - 2(x+1) = x^2 + 2x + 1,$$

$$(x+1)(x-1) = (x+1)^2,$$

$$x_1 = -1,$$

$$\underline{x \in \langle 0, \infty \rangle} \implies$$

$$(x+1)^2 - 2(x+1) = x^2 - 2x + 1,$$

$$(x+1)(x-1) = (x-1)^2,$$

$$x_2 = 1,$$

Rješenja su $x_1 = -1$, $x_2 = 1$.