

Zadatak 20. Dana je funkcija $f(x) = \frac{x-1}{x+1}$, $x \neq -1$. Odredi kompoziciju $f^n(x) = (f \circ f \circ \dots \circ f)(x)$.

Rješenje.

Imamo redom:

$$(f \circ f)(x) = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = \frac{x-1-x-1}{x-1+x+1} = -\frac{1}{x},$$

$$(f \circ f \circ f)(x) = \frac{-\frac{1}{x} - 1}{-\frac{1}{x} + 1} = -\frac{x+1}{x-1} = -\frac{1}{f(x)},$$

$$f^4(x) = \frac{-\frac{x+1}{x-1} - 1}{-\frac{x+1}{x-1} + 1} = \frac{-x-1-x+1}{-x-1+x-1} = x,$$

$$f^5(x) = \frac{x-1}{x+1} = f(x),$$

$$f^6(x) = -\frac{1}{x}$$

te izvodimo poopćenje:

$$f^n(x) = \begin{cases} f(x), & n = 4k - 3, \\ -\frac{1}{x}, & n = 4k - 2, \\ -\frac{1}{f(x)}, & n = 4k - 1, \\ x, & n = 4k, k \in \mathbf{N}. \end{cases}$$