



Zadatak 26. Odredi $f^{-1}(x)$ ako je funkcija f zadana uvjetom:

$$\begin{array}{ll} 1) f\left(\frac{1}{x}\right) = \frac{2-x}{x+2}; & 2) f\left(\frac{x}{2x-1}\right) = \frac{1}{x}; \\ 3) f\left(\frac{x+1}{x-1}\right) = \frac{x-1}{x+1}; & 4) f\left(\frac{2x}{3x+2}\right) = \frac{x}{x+1}. \end{array}$$

Rješenje. 1) $f\left(\frac{1}{x}\right) = \frac{\frac{2}{x} - 1}{1 + \frac{2}{x}} \implies f(x) = \frac{2x-1}{2x+1},$

$$x = \frac{2y-1}{2y+1},$$

$$x(2y+1) = 2y-1,$$

$$2y(x-1) = -x-1,$$

$$y = \frac{x+1}{2(1-x)},$$

$$f^{-1}(x) = \frac{x+1}{2(1-x)}, \quad x \neq -\frac{1}{2}.$$

2) $f\left(\frac{x}{2x-1}\right) = \frac{1}{x}$, stavimo $t = \frac{x}{2x-1}$ tada je $t(2x-1) = x$, $2xt - t = x$,
 $x(2t-1) = t$, $x = \frac{t}{2t-1}$, odnosno $\frac{1}{x} = \frac{2t-1}{t}$ te je

$$f(x) = \frac{2x-1}{x}, \quad x \neq 0$$

Sada odredimo f^{-1} :

$$x = \frac{2y-1}{y}, \quad xy = 2y-1, \quad y(2-x) = 1 \implies y = \frac{1}{2-x}$$

dakle

$$f^{-1}(x) = \frac{1}{2-x}.$$

3) $f\left(\frac{x+1}{x-1}\right) = \frac{x-1}{x+1} \implies f(x) = \frac{1}{x};$

$$x = \frac{1}{y} \implies y = \frac{1}{x}$$

$$f^{-1}(x) = \frac{1}{x}, \quad x \neq 0.$$

4) $f\left(\frac{2x}{3x+2}\right) = \frac{x}{x+1};$

Stavimo $t = \frac{2x}{3x+2}$ tada je

$$3xt + 2t = 2x, \quad x(2-3t) = 2t, \quad x = \frac{2t}{2-3t}$$

te je

$$f(t) = \frac{\frac{2t}{2-3t}}{\frac{2t}{2-3t} + 1} = \frac{2t}{2t + 2 - 3t} = \frac{2t}{2-t} \implies f(x) = \frac{2x}{2-x};$$

Potražimo sada f^{-1} :

$$x = \frac{2y}{2-y}, \quad x(2-y) = 2y, \quad 2x - xy = 2y, \quad 2y + xy = 2x, \quad y = \frac{2x}{2+x}.$$

$$\text{Dakle } f^{-1}(x) = \frac{2x}{x+2}, \quad x \neq -2.$$