

Zadatak 32. Izračunaj sljedeće limese:

$$1) \lim_{x \rightarrow 1} \frac{x^m - 1}{x^n - 1}, \quad m, n \in \mathbf{N};$$

$$2) \lim_{x \rightarrow a} \frac{x^2 - (a+1)x + a}{x^3 - a^3};$$

$$3) \lim_{x \rightarrow \infty} \frac{(x+1)^5 + (x+2)^5 + \dots + (x+n)^5}{x^5 + n^5}.$$

Rješenje.

1)

$$\lim_{x \rightarrow 1} \frac{x^m - 1}{x^n - 1} = \frac{(x-1)(x^{m-1} + x^{m-2} + \dots + x + 1)}{(x-1)(x^{n-1} + x^{n-2} + \dots + x + 1)} = \frac{m}{n}.$$

2)

$$\begin{aligned} \lim_{x \rightarrow a} \frac{x^2 - (a+1)x + a}{x^3 - a^3} &= \lim_{x \rightarrow a} \frac{x^2 - ax - x + a}{(x-a)(x^2 + ax + a^2)} \\ &= \lim_{x \rightarrow a} \frac{(x-a)(x-1)}{(x-a)(x^2 + ax + a^2)} \\ &= \lim_{x \rightarrow a} \frac{x-1}{x^2 + ax + a^2} = \frac{a-1}{3a^2}. \end{aligned}$$

3)

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{(x+1)^5 + (x+2)^5 + \dots + (x+n)^5}{x^5 + n^5} &= \lim_{x \rightarrow \infty} \frac{(x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1) + \dots + (x^5 + 5nx^4 + 10n^2x^3 + 10n^3x^2 + 5n^4x + n^5)}{x^5 + n^5} \\ &= \lim_{x \rightarrow \infty} \frac{\left(1 + 5\frac{1}{x} + 10\frac{1}{x^2} + 10\frac{1}{x^3} + 5\frac{1}{x^4} + \frac{1}{x^5}\right) + \dots + \left(1 + 5\frac{n}{x} + 10\frac{n^2}{x^2} + 10\frac{n^3}{x^3} + 5\frac{n^4}{x^4} + \frac{n^5}{x^5}\right)}{1 + \frac{n^5}{x^5}} \\ &= \frac{n \cdot 1}{1} = n. \end{aligned}$$