Zadatak 33. Izračunaj sljedeće limese:

1) $\lim\limits_{x\to 0} \dfrac{\sqrt[n]{x+1}-1}{x}$;

2) $\lim\limits_{x\to 1} \dfrac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1}$, $m, n \in \mathbf{N}$.

Rješenje. **1)**

$$\lim_{x\to 0} \frac{\sqrt[n]{x+1}-1}{x} = \{t = x+1,\ x = t-1\} = \lim_{t\to 1} \frac{t^{\frac{1}{n}}-1}{t-1} = \lim_{t\to 1} \frac{t^{\frac{1}{n}}-1}{\left(t^{\frac{1}{n}}\right)^{n}-1}$$

$$= \lim_{t\to 1} \frac{t^{\frac{1}{n}}-1}{\left(t^{\frac{1}{n}}-1\right)\left(\left(t^{\frac{1}{n}}\right)^{n-1}+\left(t^{\frac{1}{n}}\right)^{n-2}+\ldots+t+1\right)}$$

$$= \lim_{t\to 1} \frac{1}{\left(t^{\frac{1}{n}}\right)^{n-1}+\left(t^{\frac{1}{n}}\right)^{n-2}+\ldots+t+1} = \frac{1}{n\cdot 1} = \frac{1}{n}.$$

2)

$$\lim_{x\to 1} \frac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1} = \{t = \sqrt[m\cdot n]{x},\ x = t^{m\cdot n},\ \ x\to 1,\ t\to 1\}$$

$$= \lim_{t\to 1} \frac{t^m-1}{t^n-1} = \lim_{t\to 1} \frac{(t-1)(t^{m-1}+t^{m-2}+\ldots+t+1)}{(t-1)(t^{n-1}+t^{n-2}+\ldots+t+1)}$$

$$= \lim_{t\to 1} \frac{t^{m-1}+t^{m-2}+\ldots+t+1}{t^{n-1}+t^{n-2}+\ldots+t+1} = \frac{m}{n}.$$