

Zadatak 34. Dokaži da vrijedi $\lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a} + \sqrt{x-a}}{\sqrt{x^2 - a^2}} = \frac{1}{\sqrt{2a}}$.

Rješenje.

$$\begin{aligned} \lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a} + \sqrt{x-a}}{\sqrt{x^2 - a^2}} &= \lim_{x \rightarrow a} \frac{(\sqrt{x} - \sqrt{a} + \sqrt{x-a})(\sqrt{x} + \sqrt{a})}{\sqrt{x^2 - a^2}(\sqrt{x} + \sqrt{a})} \\ &= \lim_{x \rightarrow a} \frac{x - a + \sqrt{x-a}(\sqrt{x} + \sqrt{a})}{\sqrt{x^2 - a^2}(\sqrt{x} + \sqrt{a})} = \lim_{x \rightarrow a} \frac{\sqrt{x-a} + \sqrt{x} + \sqrt{a}}{\sqrt{x+a}(\sqrt{x} + \sqrt{a})} \\ &= \frac{2\sqrt{a}}{\sqrt{2a}(2\sqrt{a})} = \frac{1}{\sqrt{2a}}. \end{aligned}$$

Oдавде slijedi rezultat.