

Zadatak 35. Izračunaj sljedeće limese:

- 1) $\lim_{x \rightarrow -8} \frac{\sqrt{1-x} - 3}{2 + \sqrt[3]{x}};$
- 2) $\lim_{x \rightarrow \infty} x^{\frac{3}{2}} (\sqrt{x^3 + 1} - \sqrt{x^3 - 1});$
- 3) $\lim_{x \rightarrow \infty} x(\sqrt{x^2 + 1} - x);$
- 4) $\lim_{x \rightarrow \infty} (\sqrt{(x+a)(x+b)} - x).$

Rješenje. 1)

$$\begin{aligned} \lim_{x \rightarrow -8} \frac{\sqrt{1-x} - 3}{2 + \sqrt[3]{x}} \cdot \frac{\sqrt{1-x} + 3}{\sqrt{1-x} + 3} &= \lim_{x \rightarrow -8} \frac{1-x-9}{(2+\sqrt[3]{x})(\sqrt{1-x}+3)} \\ &= \lim_{x \rightarrow -8} \frac{-(2+\sqrt[3]{x})(4-2\sqrt[3]{x}+(\sqrt[3]{x})^2)}{(2+\sqrt[3]{x})(\sqrt{1-x}+3)} \\ &= \lim_{x \rightarrow -8} \frac{-(4-2\sqrt[3]{x}+(\sqrt[3]{x})^2)}{\sqrt{1-x}+3} = \frac{-(4-4+4)}{3+3} = -2 \end{aligned}$$

2)

$$\begin{aligned} \lim_{x \rightarrow \infty} x^{\frac{3}{2}} (\sqrt{x^3 + 1} - \sqrt{x^3 - 1}) \cdot \frac{\sqrt{x^3 + 1} + \sqrt{x^3 - 1}}{\sqrt{x^3 + 1} + \sqrt{x^3 - 1}} &= \lim_{x \rightarrow \infty} x^{\frac{3}{2}} \frac{x^3 + 1 - x^3 + 1}{\sqrt{x^3 + 1} + \sqrt{x^3 - 1}} \\ &= \lim_{x \rightarrow \infty} \frac{2x^{\frac{3}{2}}}{\sqrt{x^3 + 1} + \sqrt{x^3 - 1}} = \lim_{x \rightarrow \infty} \frac{2}{\sqrt{1 + \frac{1}{x^3}} + \sqrt{1 - \frac{1}{x^3}}} = \frac{2}{2} = 1 \end{aligned}$$

3)

$$\begin{aligned} \lim_{x \rightarrow \infty} x(\sqrt{x^2 + 1} - x) \cdot \frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1} + x} &= \lim_{x \rightarrow \infty} x \frac{x^2 + 1 - x^2}{\sqrt{x^2 + 1} + x} \\ &= \lim_{x \rightarrow \infty} x \frac{x^2 + 1 - x^2}{\sqrt{x^2 + 1} + x} = \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1} + x} \\ &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{x^2}} + 1} = \frac{1}{1+1} = \frac{1}{2} \end{aligned}$$

$$\frac{1}{2};$$

4)

$$\begin{aligned} & \lim_{x \rightarrow \infty} \left(\sqrt{(x+a)(x+b)} - x \right) \cdot \frac{\sqrt{(x+a)(x+b)} + x}{\sqrt{(x+a)(x+b)} + x} = \lim_{x \rightarrow \infty} \frac{(x+a)(x+b) - x^2}{\sqrt{(x+a)(x+b)} + x} \\ &= \lim_{x \rightarrow \infty} \frac{x^2 + x(a+b) + ab - x^2}{\sqrt{(x+a)(x+b)} + x} = \lim_{x \rightarrow \infty} \frac{x(a+b) + ab}{\sqrt{(x+a)(x+b)} + x} \\ &= \lim_{x \rightarrow \infty} \frac{a + b + \frac{ab}{x}}{\sqrt{\left(1 + \frac{a}{x}\right)\left(1 + \frac{b}{x}\right)} + 1} = \frac{a + b}{2}. \end{aligned}$$