Zadatak 36. Izračunaj sljedeće limese:

1) $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$;

2) $\lim\limits_{x\to 1}(1+\sin \pi x)^{\operatorname{ctg} \pi x}$;

3) $\lim\limits_{x\to 0}(1+x^2)^{\operatorname{ctg}^2 x}$;

4) $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{x^2}}$.

Rješenje. **1)** Vrijedi

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} = \lim_{x\to 0}\left[(1+\sin x)^{\frac{1}{\sin x}}\right]^{\lim\limits_{x\to 0}\frac{\sin x}{x}} = e^1 = e.$$

2)

$$\lim_{x\to 1}(1+\sin \pi x)^{\operatorname{ctg} \pi x} = \lim_{x\to 1}\left[(1+\sin \pi x)^{\frac{1}{\sin \pi x}}\right]^{\cos \pi x} = e^{-1}.$$

3)

$$\begin{aligned}\lim_{x\to 0}(1+x^2)^{\operatorname{ctg}^2 x} &= \lim_{x\to 0}\left[(1+x^2)^{\frac{1}{x^2}}\right]^{x^2\cdot\frac{\cos^2 x}{\sin^2 x}}\\ &= \lim_{x\to 0}\left[(1+x^2)^{\frac{1}{x^2}}\right]^{\lim\limits_{x\to 0}\frac{1}{\frac{\sin^2 x}{x^2}}\cdot\lim\limits_{x\to 0}\cos^2 x}\\ &= e^{1\cdot 1} = e\end{aligned}$$

4)

$$\begin{aligned}\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}} &= \lim_{x\to 0}(1-2\sin^2\tfrac{x}{2})^{\frac{1}{x^2}} = \lim_{x\to 0}\left[(1-2\sin^2\tfrac{x}{2})^{\frac{1}{-2\sin^2\frac{x}{2}}}\right]^{-2\sin^2\frac{x}{2}\cdot\frac{1}{x^2}}\\ &= \lim_{x\to 0}\left[(1-2\sin^2\tfrac{x}{2})^{\frac{1}{2\sin^2\frac{x}{2}}}\right]^{-\frac{1}{2}\cdot\frac{\sin^2\frac{x}{2}}{\frac{x^2}{4}}} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}.\end{aligned}$$