

Rješenja zadataka 5.1

Zadatak 1. Izračunaj donju i gornju sumu za integral funkcije $f(x) = x$ na intervalu $[0, a]$ i pomoću njih odredi vrijednost tog integrala.

Rješenje. Širina pravokutnika je $\frac{a}{n} \cdot \underset{\text{donja}}{P_n} < P < \underset{\text{gornja}}{P_n}$.

Gornja suma

$$\begin{aligned} P_n &= \frac{a}{n} \cdot \frac{a}{n} + \frac{a}{n} \cdot \frac{2a}{n} + \frac{a}{n} \cdot \frac{3a}{n} + \dots + \frac{a}{n} \cdot \frac{na}{n} = \frac{a^2}{n^2} \cdot (1 + 2 + 3 + \dots + n) \\ &= \frac{a^2}{n^2} \cdot \frac{n(n+1)}{2} = \frac{a^2}{2} \cdot \frac{n+1}{n}, \quad \lim_{n \rightarrow \infty} P_n = \frac{a^2}{2}; \end{aligned}$$

Donja suma

$$\begin{aligned} p_n &= \frac{a}{n} \cdot \frac{a}{n} + \frac{a}{n} \cdot \frac{2a}{n} + \dots + \frac{a}{n} \cdot \frac{(n-1)a}{n} = \frac{a^2}{n^2} (1 + 2 + 3 + \dots + (n-1)) \\ &= \frac{a^2}{n^2} \cdot \frac{(n-1)n}{2} = \frac{a^2}{2} \cdot \frac{n-1}{n}, \quad \lim_{n \rightarrow \infty} p_n = \frac{a^2}{2}, \\ &\implies P = \frac{a^2}{2}. \end{aligned}$$