Zadatak 2. Površina lika ispod pravca $y = x$, a nad intervalom $[a, b]$ $(0 < a < b)$ iznosi $\frac{b^2 - a^2}{2}$. Dokaži to! Izračunaj površinu ispod pravca $y = kx + l$ nad intervalom $[a, b]$ (pretpostavi da pravac nad tim intervalom leži iznad x-osi).

Rješenje. $P_{[a,b]} = P_{[0,b]} - P_{[0,a]} = \frac{b^2}{2} - \frac{a^2}{2} = \frac{b^2 - a^2}{2}$.

$f(x) = kx + l$, $[a, b]$, $0 < a < b$, $P = P_b - P_a$,

Gornja suma

$$\begin{aligned}
P_n &= \frac{b}{n} \cdot \left(k \cdot \frac{b}{n} + l + k \cdot \frac{2b}{n} + l + k \cdot \frac{3b}{n} + l + \ldots + k \cdot \frac{n \cdot b}{n} + l\right) \\
&= \frac{b^2}{n^2} k(1 + 2 + 3 + \ldots + n) + \frac{b}{n} \cdot nl \\
&= \frac{b^2}{n^2} \cdot k \cdot \frac{n(n+1)}{2} + bl = \frac{b^2 k}{2} \cdot \frac{n+1}{n} + bl, \quad \lim_{n\to\infty} P_n = \frac{b^2 k}{2} + bl;
\end{aligned}$$

Donja suma

$$p_n = \frac{b^2 k}{2} \cdot \frac{n-1}{n} + bl, \quad \lim_{n\to\infty} p_n = \frac{b^2 k}{2} + bl,$$

$\Longrightarrow$ $P_b = \frac{b^2 k}{2} + bl$. Analogno, $P_a = \frac{a^2 k}{2} + al$ $\Longrightarrow$ $P = \frac{b^2 - a^2}{2} \cdot k + (b - a)l$.