Zadatak 3. Površina lika omeđenog lukom parabole $y = x^2$, pravcima $x = a$ i $x = b$ $(0 < a < b)$, te s osi apscisa jednaka je $P = \dfrac{b^3 - a^3}{3}$. Dokaži!

Rješenje. $f(x) = x^2$, $[a, b]$, $0 < a < b$, $P = P_b - P_a$

Gornja suma

$$P_n = \frac{b}{n} \cdot \left(\frac{b^2}{n^2} + \frac{2^2 b^2}{n^2} + \frac{3^2 b^2}{n^2} + \ldots + \frac{n^2 b^2}{n^2}\right) = \frac{b^3}{n^3}(1 + 2^2 + 3^2 + \ldots + n^2)$$

$$P_n = \frac{b^3}{n^3}\frac{n(n+1)(2n+1)}{6}, \quad \lim_{n\to\infty} P_n = \frac{b^3}{3};$$

Donja suma

$$p_n = \frac{b}{n}\left(\frac{b^2}{n^2} + \frac{2^2 b^2}{n^2} + \ldots + \frac{(n-1)^2 b^2}{n^2}\right) = \frac{b^3}{n^3}(1 + 2^2 + 3^2 + \ldots + (n-1)^2)$$

$$p_n = \frac{b^3}{n^3}\frac{(n-1)\cdot n\cdot(2n-1)}{6}, \quad \lim_{n\to\infty} p_n = \frac{b^3}{3};$$

$$\Longrightarrow \quad P_b = \frac{b^3}{3}.$$

Analogno $P_a = \dfrac{a^3}{3} \quad \Longrightarrow \quad P = \dfrac{b^3 - a^3}{3}$.