

Zadatak 3. Provjeri sljedeće jednakosti:

$$1) \int \frac{(1 + \sqrt{x})^2}{\sqrt{x}} dx = \frac{2}{3}(1 + \sqrt{x})^3 + C;$$

$$2) \int \frac{x-1}{x+1} dx = x - 2 \ln|x+1| + C;$$

$$3) \int \frac{x+1}{x^2+2x+2} dx = \frac{1}{2} \ln(x^2+2x+2) + C;$$

$$4) \int \frac{e^{2x}}{e^{2x}+3} dx = \frac{1}{2} \ln(e^{2x}+3) + C;$$

$$5) \int \frac{dx}{\sqrt{x}(1-\sqrt{x})} = \ln \frac{C}{(1-\sqrt{x})^2};$$

$$6) \int \frac{dx}{x^2-4} = \frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + C.$$

Rješenje.

$$1) F'(x) = \frac{2}{3} \cdot 3(1 + \sqrt{x})^2 \cdot \frac{1}{2\sqrt{x}} = \frac{(1 + \sqrt{x})^2}{\sqrt{x}} = f(x);$$

$$2) F'(x) = 1 - \frac{2}{x+1} = \frac{x+1-2}{x+1} = \frac{x-1}{x+1} = f(x);$$

$$3) F'(x) = \frac{1}{2} \cdot \frac{2x+2}{x^2+2x+2} = \frac{x+1}{x^2+2x+2} = f(x);$$

$$4) F'(x) = \frac{1}{2} \cdot \frac{1}{e^{2x}+3} \cdot 2e^{2x} = \frac{e^{2x}}{e^{2x}+3} = f(x);$$

$$5) F'(x) = (1 - \sqrt{x})^2 \cdot \frac{-2}{(1 - \sqrt{x})^3} \cdot \frac{-1}{2\sqrt{x}} = \frac{1}{(1 - \sqrt{x})\sqrt{x}} = f(x);$$

$$6) F'(x) = \frac{1}{4} \frac{x+2}{x-2} \cdot \frac{x+2-x+2}{(x+2)^2} = \frac{1}{x^2-4} = f(x).$$