Zadatak 4. Provjeri sljedeće jednakosti:

1) $\displaystyle\int \frac{1}{1+\cos 3x}dx = \frac{1-\cos 3x}{3\sin 3x} + C\,;$

2) $\displaystyle\int \cos^4 x \sin x\, dx = -\tfrac{1}{5}\cos^5 x + C\,;$

3) $\displaystyle\int (\cos x - \sin x)^2 dx = x + \tfrac{1}{2}\cos 2x + C\,;$

4) $\displaystyle\int e^{2\sin 3x}\cdot \cos 3x\, dx = \tfrac{1}{6}e^{2\sin 3x} + C\,.$

Rješenje. **1)** $\displaystyle F'(x) = \frac{3\sin 3x\cdot 3\sin 3x-(1-\cos 3x)9\cos 3x}{(3\sin 3x)^2} = \frac{9\sin^2 3x+9\cos^2 3x-9\cos 3x}{9\sin^2 3x}$

$\displaystyle = \frac{9(1-\cos 3x)}{9(1-\cos^2 3x)} = \frac{1-\cos 3x}{(1-\cos 3x)(1+\cos 3x)} = \frac{1}{1+\cos 3x} = f(x)\,;$

2) $\displaystyle F'(x) = -\frac{1}{5}5\cos^4 x(-\sin x) = \cos^4 x\sin x = f(x)\,;$

3) $\displaystyle F'(x) = 1+\frac{1}{2}(-2\sin 2x) = \cos^2 x+\sin^2 x-\sin 2x = \cos^2 x-2\sin x\cos x+$
$\displaystyle \sin^2 x = (\cos x-\sin x)^2 = f(x)\,;$

4) $\displaystyle F'(x) = \frac{1}{6}\cdot e^{2\sin 3x}\cdot 2\cdot 3\cdot \cos 3x = e^{2\sin 3x}\cos 3x = f(x)\,.$