Zadatak 6. Za danu funkciju f nađi primitivnu funkciju F uz dani uvjet:

1) $f(x) = 1 - \frac{1}{2}x$, $F(0) = -1$;

2) $f(x) = 2x - 3$, $F(1) = -3$;

3) $f(x) = \frac{1}{\sqrt{x-1}}$, $F(2) = 2$;

4) $f(x) = \sin \pi x$, $F(1) = \pi$;

5) $f(x) = \frac{1}{x+1}$, $F(-2) = 0$;

6) $f(x) = x^2 - x + 1$, $F(1) = 0$.

Rješenje. **1)** $F(x) = x - \frac{1}{4}x^2 + C$, $F(0) = C = -1 \implies F(x) = x - \frac{1}{4}x^2 - 1 = -\left(\frac{1}{4}x^2 - x + 1\right) \implies F(x) = -\left(\frac{1}{2}x - 1\right)^2$;

2) $F(x) = x^2 - 3x + C$, $F(1) = C - 2 = -3 \implies C = -1 \implies F(x) = x^2 - 3x - 1$;

3) $F(x) = 2\sqrt{x-1} + C$, $F(2) = 2 + C = 2 \implies C = 0 \implies F(x) = 2\sqrt{x-1}$;

4) $F(x) = \frac{-1}{\pi}\cos \pi x + C$, $F(1) = -\frac{1}{\pi}\cos \pi + C = \frac{1}{\pi} + C = \pi$

$\implies C = \pi - \frac{1}{\pi} = \frac{\pi^2 - 1}{\pi} \implies F(x) = -\frac{1}{\pi}\cos \pi x + \frac{\pi^2 - 1}{\pi}$;

5) $F(x) = \ln|x+1| + C$, $F(-2) = \ln|-2+1| + C = C = 0 \implies F(x) = \ln|x+1|$;

6) $F(x) = \frac{x^3}{3} - \frac{x^2}{2} + x + C$, $F(1) = \frac{1}{3} - \frac{1}{2} + 1 + C = 0$,

$\frac{2 - 3 + 6}{6} + C = 0 \implies C = -\frac{5}{6} \implies F(x) = \frac{x^3}{3} - \frac{x^2}{2} + x - \frac{5}{6}$.