

Zadatak 10. Trokut ima vrhove u točkama $(0, 0)$, $(a, 0)$ i (c, v) ($0 < c < a$). Pomoću integralnog računa izračunaj njegovu površinu.

Rješenje. $A(0, 0)$, $(x_1, y_1) = B(a, 0)$, $(x_2, y_2) = C(c, v)$

$$y - 0 = \frac{v - 0}{c - a}(x - a), \quad y = \frac{v}{c - a}x - \frac{av}{c - a}.$$

$$\begin{aligned} P &= \int_0^c \frac{v}{c} x dx + \int_c^a \left(\frac{v}{c - a} x - \frac{av}{c - a} \right) dx = \frac{v}{2c} x^2 \Big|_0^c + \left(\frac{v}{2(c - a)} x^2 - \frac{av}{c - a} x \right) \Big|_c^a \\ &= \frac{vc}{2} + \frac{va^2}{2(c - a)} - \frac{vc^2}{2(c - a)} + \frac{vac}{c - a} - \frac{a^2v}{c - a} \\ &= \frac{vc^2 - vac + va^2 - vc^2 + 2vac - 2va^2}{2(c - a)} = \frac{vac - va^2}{2(c - a)} \\ &= \frac{va(c - a)}{2(c - a)} = \frac{va}{2}. \end{aligned}$$