Rješenja zadataka 5.3

Zadatak 1. Odredi:

1) $\int 8x^3 dx$;

2) $\int x^{\frac{2}{3}} dx$;

3) $\int 2x^{-\frac{1}{2}} dx$;

4) $\int \frac{dx}{x^{3/4}}$;

5) $\int \pi^2 dx$;

6) $\int (-4x) dx$;

7) $\int (e-\sqrt{3}) dx$;

8) $\int (2\pi - x) dx$;

9) $\int x\sqrt{3} dx$;

10) $\int x\sqrt{x} dx$;

11) $\int x\sqrt[3]{x} dx$;

12) $\int x\sqrt[3]{x\sqrt{x}} dx$;

13) $\int x^2\sqrt{x} dx$;

14) $\int \frac{dx}{\sqrt[4]{x^3}}$;

15) $\int \left(-\frac{\ln \pi}{x^2}\right) dx$.

Rješenje.

1) $\int 8x^3 dx = 8\int x^3 dx = 8\frac{x^4}{4} + C = 2x^4 + C$;

2) $\int x^{\frac{2}{3}} dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + C = \frac{3}{5}x^{\frac{5}{3}} + C = \frac{3}{5}x\sqrt[3]{x^2} + C$;

3) $\int 2x^{-\frac{1}{2}} dx = 2\int x^{-\frac{1}{2}} dx = 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = 4\sqrt{x} + C$;

4) $\int \frac{dx}{x^{\frac{3}{4}}} = \int x^{-\frac{3}{4}} dx = \frac{x^{\frac{1}{4}}}{\frac{1}{4}} + C = 4\sqrt[4]{x} + C$;

5) $\int \pi^2 dx = \pi^2 \int dx = \pi^2 x + C$;

6) $\int -4x\, dx = -4\int x\, dx = -4 \cdot \frac{x^2}{2} + C = -2x^2 + C$;

7) $\int (e-\sqrt{3}) dx = (e-\sqrt{3})\int dx = (e-\sqrt{3})x + C$;

8) $\int (2\pi - x) dx = \int 2\pi\, dx - \int x\, dx = 2\pi \int dx - \int x\, dx = 2\pi x - \frac{1}{2}x^2 + C$;

9) $\int x\sqrt{3} dx = \sqrt{3}\int x\, dx = \sqrt{3} \cdot \frac{x^2}{2} + C = \frac{\sqrt{3}}{2}x^2 + C$;

10) $\int x\sqrt{x} dx = \int x^{\frac{3}{2}} dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C = \frac{2}{5}x^{\frac{5}{2}} + C = \frac{2}{5}x^2\sqrt{x} + C$;

11) $\int x\sqrt[3]{x}dx = \int x^{\frac{4}{3}}dx = \frac{x^{\frac{7}{3}}}{\frac{7}{3}} + C = \frac{3}{7}x^2\sqrt[3]{x} + C;$

12) $\int x\sqrt[3]{x\sqrt{x}}dx = \int x^{\frac{9}{6}}dx = \int x^{\frac{3}{2}}dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C = \frac{2}{5}x^2\sqrt{x} + C;$

13) $\int x^2\sqrt{x}dx = \int x^{\frac{5}{2}}dx = \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + C = \frac{2}{7}x^3\sqrt{x} + C;$

14) $\int \frac{dx}{\sqrt[4]{x^3}} = \int x^{-\frac{3}{4}}dx = \frac{x^{\frac{1}{4}}}{\frac{1}{4}} + C = 4\sqrt[4]{x} + C;$

15) $\int \left(-\frac{\ln\pi}{x^2}\right)dx = -\ln\pi\int\left(\frac{dx}{x^2}\right) = -\ln\pi\int x^{-2}\,dx = -\ln\pi\cdot\frac{x^{-1}}{-1} + C =$
$\ln\pi\cdot\frac{1}{x} + C = \frac{\ln\pi}{x} + C.$