Zadatak 2. Odredi:

1) $\displaystyle\int (1 - 2x)dx$;

2) $\displaystyle\int (x^2 - 3)dx$;

3) $\displaystyle\int 3\pi dx$;

4) $\displaystyle\int (x - 1)^2 dx$;

5) $\displaystyle\int \frac{1}{\sqrt[3]{x^2}}dx$;

6) $\displaystyle\int (1 - x)\sqrt{x}\,dx$;

7) $\displaystyle\int (x^{\frac{1}{2}} - x^{\frac{3}{2}})\sqrt[4]{x}dx$;

8) $\displaystyle\int (2\sqrt{x} - 1)^2 dx$;

9) $\displaystyle\int \frac{x - 1}{\sqrt{x} + 1}dx$;

10) $\displaystyle\int \frac{x + 1}{\sqrt[3]{x} + 1}dx$;

11) $\displaystyle\int \frac{(\sqrt{x} + \sqrt[3]{x})^2}{\sqrt{x}}dx$;

12) $\displaystyle\int \frac{x + \sqrt{x}}{\sqrt[3]{x}}dx$.

Rješenje.

1) $\displaystyle\int (1 - 2x)dx = \int dx - 2\int x\,dx = x - 2\frac{x^2}{2} + C = x - x^2 + C$;

2) $\displaystyle\int (x^2 - 3)dx = \int x^2 dx - 3\int dx = \frac{x^3}{3} - 3x + C$;

3) $\displaystyle\int 3\pi dx = 3\pi \int dx = 3\pi x + C$;

4) $\displaystyle\int (x - 1)^2 dx = \int (x^2 - 2x + 1)dx = \int x^2 dx - 2\int x\,dx + \int dx =$

$\dfrac{x^3}{3} - 2\dfrac{x^2}{2} + x + C = \dfrac{x^3}{3} - x^2 + x + C = \dfrac{x^3}{3} - x^2 + x - \dfrac{1}{3} + C_1 = \dfrac{1}{3}(x - 1)^3 + C_1$;

5) $\displaystyle\int \frac{1}{\sqrt[3]{x^2}}dx = \int x^{-\frac{2}{3}}dx = \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C = 3\sqrt[3]{x} + C$;

6) $\displaystyle\int (1 - x)\sqrt{x}dx = \int \sqrt{x}dx - \int x\sqrt{x}\,dx = \int x^{\frac{1}{2}}dx - \int x^{\frac{3}{2}}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} -$

$\dfrac{x^{\frac{5}{2}}}{\frac{5}{2}} + C = \dfrac{2}{3}x\sqrt{x} - \dfrac{2}{5}x^2\sqrt{x} + C = 2x\sqrt{x}\Big(\dfrac{1}{3} - \dfrac{1}{5}x\Big) + C$;

7) $\displaystyle\int (x^{\frac{1}{2}} - x^{\frac{3}{2}})\sqrt[4]{x}dx = \int x^{\frac{1}{2}}x^{\frac{1}{4}}dx - \int x^{\frac{3}{2}}x^{\frac{1}{4}}dx = \int x^{\frac{3}{4}}dx - \int x^{\frac{7}{4}}dx =$

$\dfrac{x^{\frac{7}{4}}}{\frac{7}{4}} - \dfrac{x^{\frac{11}{4}}}{\frac{11}{4}} + C = \dfrac{4}{7}x\sqrt[4]{x^3} - \dfrac{4}{11}x^2\sqrt[4]{x^3} + C = 4x\sqrt[4]{x^3}\Big(\dfrac{1}{7} - \dfrac{1}{11}x\Big) + C$;

8) $\displaystyle\int (2\sqrt{x} - 1)^2 dx = \int (4x - 4\sqrt{x} + 1)dx = 4\int x\,dx - 4\int x^{\frac{1}{2}}dx + \int dx =$

$4\dfrac{x^2}{2} - 4\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + x + C = 2x^2 - \dfrac{8}{3}x\sqrt{x} + x + C$;

9) $\displaystyle\int \frac{x - 1}{\sqrt{x} + 1}dx = \int \frac{(\sqrt{x} - 1)(\sqrt{x} + 1)}{\sqrt{x} + 1}dx = \int (\sqrt{x} - 1)dx = \int x^{\frac{1}{2}}dx -$

$\displaystyle\int dx = \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} - x + C = \dfrac{2}{3}x\sqrt{x} - x + C$;

10) $\displaystyle\int \frac{x+1}{\sqrt[3]{x}+1}dx = \int \frac{(\sqrt[3]{x}+1)(\sqrt[3]{x^2}-\sqrt[3]{x}+1)}{\sqrt[3]{x}+1}dx = \int (x^{\frac{2}{3}}-x^{\frac{1}{3}}+1)dx =$

$\displaystyle\int x^{\frac{2}{3}}dx - \int x^{\frac{1}{3}}dx + \int dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} - \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + x + C = \frac{3}{5}x\sqrt[3]{x^2} - \frac{3}{4}x\sqrt[3]{x} + x + C;$

11) $\displaystyle\int \frac{(\sqrt{x}+\sqrt[3]{x})^2}{\sqrt{x}}dx = \int \frac{x + 2\sqrt[6]{x^5} + \sqrt[3]{x^2}}{\sqrt{x}}dx = \int \frac{x + 2x^{\frac{5}{6}} + x^{\frac{2}{3}}}{x^{\frac{1}{2}}}dx =$

$\displaystyle\int (x^{\frac{1}{2}} + 2x^{\frac{1}{3}} + x^{\frac{1}{6}})dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\frac{x^{\frac{4}{3}}}{\frac{4}{3}} + \frac{x^{\frac{7}{6}}}{\frac{7}{6}} + C = \frac{2}{3}x\sqrt{x} + \frac{3}{2}x\sqrt[3]{x} + \frac{6}{7}x\sqrt[6]{x} + C;$

12) $\displaystyle\int \frac{x + \sqrt{x}}{\sqrt[3]{x}}dx = \int \frac{x + x^{\frac{1}{2}}}{x^{\frac{1}{3}}}dx = \int (x^{\frac{2}{3}} + x^{\frac{1}{6}})dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{x^{\frac{7}{6}}}{\frac{7}{6}} + C =$

$\dfrac{3}{5}x\sqrt[3]{x^2} + \dfrac{6}{7}x\sqrt[6]{x} + C.$