Zadatak 7. Nađi primitivnu funkciju svake od sljedećih funkcija:

1) $f(x) = \sin\frac{x}{3}\cdot\cos x$;

2) $f(x) = \cos\frac{x}{2}\cdot\sin\frac{3x}{2}$;

3) $f(x) = \dfrac{1}{\sin^2\left(2x+\frac{\pi}{3}\right)}$;

4) $f(x) = \dfrac{1}{\cos^2\left(\frac{\pi}{4}-3x\right)}$;

5) $f(x) = \sin 3x\cos 3x$;

6) $f(x) = 1+\sin(1-x)$;

7) $f(x) = \sin^2\frac{x}{3}-\cos^2\frac{x}{3}$;

8) $f(x) = \sqrt{2}\cos\left(2x+\frac{\pi}{3}\right)$;

9) $f(x) = \operatorname{tg}\frac{x}{3}\cdot\operatorname{ctg}\frac{x}{3}$;

10) $f(x) = \dfrac{\sin 2x+3\sin^2 x}{\sin x}$;

11) $f(x) = \dfrac{x+1}{x^2-x-2}$;

12) $f(x) = \dfrac{x^2-x+1}{x-1}$;

13) $f(x) = \dfrac{x^3-1}{x^2-x}$;

14) $f(x) = \dfrac{1+\sin^2 2x}{\cos^2 x}$.

Rješenje.

1) $F(x) = \int \sin\frac{x}{3}\cos x\,dx = \int \frac{1}{2}\left(\sin\left(-\frac{2}{3}x\right)+\sin\frac{4}{3}x\right)dx = \frac{1}{2}\cdot\frac{3}{2}\int\left(-\frac{2}{3}\sin\frac{2}{3}x\right)dx + \frac{1}{2}\cdot\frac{3}{4}\int\frac{4}{3}\sin\frac{4}{3}x\,dx = \frac{3}{4}\cos\frac{2}{3}x-\frac{3}{8}\cos\frac{4}{3}x+C$

2) $F(x) = \int\sin\frac{3x}{2}\cos\frac{x}{2}dx = \frac{1}{2}\int(\sin x+\sin 2x)dx = -\frac{1}{2}\cos x-\frac{1}{4}\cos 2x + C$;

3) $F(x) = \int\frac{dx}{\sin^2(2x+\frac{\pi}{3})} = -\frac{1}{2}\int\frac{-2dx}{\sin^2(2x+\frac{\pi}{3})} = -\frac{1}{2}\operatorname{ctg}\left(2x+\frac{\pi}{3}\right)+C$;

4) $F(x) = \int\frac{dx}{\cos^2(\frac{\pi}{4}-3x)} = -\frac{1}{3}\int\frac{-3dx}{\cos^2(\frac{\pi}{4}-3x)} = -\frac{1}{3}\operatorname{tg}\left(\frac{\pi}{4}-3x\right)+C$;

5) $F(x) = \int\sin 3x\cos 3x\,dx = \frac{1}{2}\int\sin 6x\,dx = -\frac{1}{12}\cos 6x+C$;

6) $F(x) = \int(1+\sin(1-x))dx = x+\cos(1-x)+C$;

7) $F(x) = -\int\left(\cos^2\frac{x}{3}-\sin^2\frac{x}{3}\right)dx = -\int\cos\frac{2x}{3}dx = -\frac{3}{2}\sin\frac{2x}{3}+C$;

8) $F(x) = \sqrt{2}\int\cos\left(2x+\frac{\pi}{3}\right)dx = \frac{\sqrt{2}}{2}\sin\left(2x+\frac{\pi}{3}\right)+C$;

9) $F(x) = \int \operatorname{tg}\frac{x}{3}\operatorname{ctg}\frac{x}{3}dx = \int dx = x + C;$

10) $F(x) = \int \frac{\sin 2x + 3\sin^2 x}{\sin x}dx = \int \frac{2\sin x\cos x + 3\sin^2 x}{\sin x}dx = \int \frac{\sin x(2\cos x + 3\sin x)}{\sin x}dx = 2\int \cos x\,dx + 3\int \sin x\,dx = 2\sin x - 3\cos x + C;$

11) $F(x) = \int \frac{x+1}{x^2 - x - 2}dx = \int \frac{x+1}{(x+1)(x-2)}dx = \ln|x-2| + C;$

12) $F(x) = \int \frac{x^2 - x + 1}{x-1}dx = \int \frac{x(x-1)+1}{x-1}dx = \int \left(x + \frac{1}{x-1}\right)dx = \frac{x^2}{2} + \ln|x-1| + C;$

13) $F(x) = \int \frac{x^3 - 1}{x^2 - x}dx = \int \frac{(x-1)(x^2+x+1)}{x(x-1)}dx = \int \left(x + 1 + \frac{1}{x}\right)dx = \frac{x^2}{2} + x + \ln|x| + C;$

14) $F(x) = \int \frac{1 + \sin^2 2x}{\cos^2 x}dx = \int \frac{1 + 4\sin^2 x\cos^2 x}{\cos^2 x}dx = \int \left(\frac{1}{\cos^2 x} + 4\sin^2 x\right)dx = \operatorname{tg} x + 4\int \sin^2 x\,dx = \operatorname{tg} x + 4\int \frac{1}{2}(1 - \cos 2x)dx = \operatorname{tg} x + 2x - \int 2\cos 2x\,dx = \operatorname{tg} x + 2x - \sin 2x + C.$