Zadatak 8. Izračunaj:

1) $\int_0^1 (3x^2-x+2)dx$;

2) $\int_0^2 (2+x)^2 dx$;

3) $\int_2^3 (1-x)^2 dx$;

4) $\int_{-1}^2 (3x^2-\frac{1}{2}x+1)dx$;

5) $\int_{-1}^1 \left(1+\frac{x}{2}\right)^3 dx$;

6) $\int_0^2 x(3x+1)^2 dx$;

7) $\int_1^2 \frac{1-8x^3}{2x-1}dx$;

8) $\int_0^1 (3x-\frac{1}{2})^2(x+2)dx$.

Rješenje.

1) $\int_0^1 (3x^2-x+2)dx = \left(x^3-\frac{x^2}{2}+2x\right)\Big|_0^1 = 1-\frac{1}{2}+2 = \frac{5}{2}$;

2) $\int_0^2 (2+x)^2 dx = \int_0^2 (4+4x+x^2)dx = \left(4x+2x^2+\frac{x^3}{3}\right)\Big|_0^2 = 8+8+\frac{8}{3} = \frac{56}{3}$;

3) $\int_2^3 (1-x)^2 dx = \int_2^3 (1-2x+x^2)dx = \left(x-x^2+\frac{x^3}{3}\right)\Big|_2^3 = 3-9+9-2+4-\frac{8}{3} = \frac{7}{3}$;

4) $\int_{-1}^2 \left(3x^2-\frac{1}{2}x+1\right)dx = \left(x^3-\frac{x^2}{4}+x\right)\Big|_{-1}^2 = 8-1+2+1+\frac{1}{4}+1 = \frac{45}{4}$;

5) $\int_{-1}^1 \left(1+\frac{x}{2}\right)^3 dx = \int_{-1}^1 \left(1+\frac{3}{2}x+\frac{3}{4}x^2+\frac{1}{8}x^3\right)dx = \left(x+\frac{3}{4}x^2+\frac{1}{4}x^3+\frac{1}{32}x^4\right)\Big|_{-1}^1 = 1+\frac{3}{4}+\frac{1}{4}+\frac{1}{32}+1-\frac{3}{4}+\frac{1}{4}-\frac{1}{32} = \frac{5}{2}$;

6) $\int_0^2 x(3x+1)^2 dx = \int_0^2 x(9x^2+6x+1)dx = \int_0^2 (9x^3+6x^2+x)dx = \left(\frac{9}{4}x^4+2x^3+\frac{1}{2}x^2\right)\Big|_0^2 = \frac{9}{4}\cdot 16+2\cdot 8+\frac{1}{2}\cdot 4 = 36+16+2 = 54$;

7) $\int_0^1 \left(3x-\frac{1}{2}\right)^2(x+2)dx = \int_0^1 \left(9x^2-3x+\frac{1}{4}\right)(x+2)dx = \int_0^1 \left(9x^3-3x^2+\frac{1}{4}x+18x^2-6x+\frac{1}{2}\right)dx = \int_0^1 \left(9x^3+15x^2-\frac{23}{4}x+\frac{1}{2}\right)dx = \left(\frac{9}{4}x^4+5x^3-\frac{23}{8}x^2+\frac{1}{2}x\right)\Big|_0^1 = \frac{9}{4}+5-\frac{23}{8}+\frac{1}{2} = \frac{18+40-23+4}{8} = \frac{39}{8}$

8) $\int_1^2 \frac{1-8x^3}{2x-1}dx = \int_1^2 \frac{(1-2x)(1+2x+4x^2)}{2x-1}dx = -\int_1^2 (1+2x+4x^2)dx = -\left(x+x^2+\frac{4}{3}x^3\right)\Big|_1^2 = -\left[2+4+\frac{32}{3}-\left(1+1+\frac{4}{3}\right)\right] = -\frac{40}{3}$.