

Zadatak 9. Izračunaj:

$$1) \int_1^4 \frac{x\sqrt{x}}{\sqrt[4]{x^3}} dx;$$

$$2) \int_0^9 \frac{x-1}{\sqrt{x+1}} dx;$$

$$3) \int_0^{-4} \sqrt{(4-3x)^3} dx;$$

$$4) \int_0^{12} \frac{dx}{\sqrt{1+\frac{x}{4}}};$$

$$5) \int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}};$$

$$6) \int_{-2}^2 \frac{x^3+x^2+x+1}{x^2+1} dx.$$

Rješenje.

$$1) \int_1^4 \frac{x\sqrt{x}}{\sqrt[4]{x^3}} dx = \int_1^4 x^{1+\frac{1}{2}-\frac{3}{4}} dx = \int_1^4 x^{\frac{3}{4}} dx = \frac{x^{\frac{7}{4}}}{\frac{7}{4}} \Big|_1^4 = \frac{4}{7} x^{\frac{7}{4}} \Big|_1^4 = \frac{4}{7} x^4 \sqrt{x^3} \Big|_1^4 = \frac{4}{7} (4^4 \sqrt{64} - 1) = \frac{4}{7} (4^4 \sqrt{2^6} - 1) = \frac{4}{7} (8\sqrt{2} - 1);$$

$$2) \int_0^9 \frac{x-1}{\sqrt{x+1}} dx = \int \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}+1} dx = \int_0^9 (\sqrt{x}-1) dx = \int_0^9 (x^{\frac{1}{2}} - 1) dx = \left(\frac{2}{3} x\sqrt{x} - x \right) \Big|_0^9 = \frac{2}{3} \cdot 9 \cdot 3 - 9 = 9;$$

$$3) \int_0^{-4} \sqrt{(4-3x)^3} dx = \int_0^{-4} (4-3x)^{\frac{3}{2}} dx = -\frac{1}{3} (4-3x)^{\frac{5}{2}} \cdot \frac{2}{5} \Big|_0^{-4} = -\frac{2}{15} (16^{\frac{5}{2}} - 4^{\frac{5}{2}}) = -\frac{2}{15} (4^5 - 2^5) = -\frac{2}{15} (2^{10} - 2^5) = -132 \frac{4}{15};$$

$$4) \int_0^{12} \frac{dx}{\sqrt{1+\frac{x}{4}}} = \int_0^{12} \left(1+\frac{x}{4}\right)^{-\frac{1}{2}} dx = 4 \left(1+\frac{x}{4}\right)^{\frac{1}{2}} \cdot 2 \Big|_0^{12} = 8 \sqrt{1+\frac{x}{4}} \Big|_0^{12} = 8(\sqrt{1+3} - \sqrt{1}) = 8(2-1) = 8;$$

$$5) \int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \int_0^{13} (1+2x)^{-\frac{2}{3}} dx = \frac{1}{2} (1+2x)^{\frac{1}{3}} \cdot 3 \Big|_0^{13} = \frac{3}{2} \sqrt[3]{1+2x} \Big|_0^{13} = \frac{3}{2} (\sqrt[3]{27} - \sqrt[3]{1}) = \frac{3}{2} (3-1) = 3;$$

$$6) \int_{-2}^2 \frac{x^3+x^2+x+1}{x^2+1} dx = \int_{-2}^2 \frac{(x+1)(x^2+1)}{x^2+1} dx = \left(\frac{x^2}{2} + x \right) \Big|_{-2}^2 = \frac{4}{2} + 2 - \frac{4}{2} + 2 = 4.$$