

Zadatak 10. Izračunaj:

$$1) \int_{\pi}^{2\pi} \cos \frac{x}{6} dx; \quad 2) \int_0^{\pi/8} (1 + \cos 2x) dx;$$

$$3) \int_{\pi/3}^{\pi} \sin \left(\frac{x}{2} - \frac{\pi}{6} \right) dx;$$

$$4) \int_0^{\pi/2} \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2 dx;$$

$$5) \int_0^{\pi/2} \cos^2 \frac{x}{2} dx; \quad 6) \int_0^{\pi/2} \sin x \cdot \cos 3x dx;$$

$$7) \int_0^{\pi/18} (\cos x \cos 2x - \sin x \sin 2x) dx;$$

$$8) \int_{\pi/3}^{\pi/2} \cos^4 \left(x - \frac{\pi}{12} \right) dx;$$

$$9) \int_{\pi/6}^{\pi/4} (\operatorname{tg} x + \operatorname{ctg} x)^{-1} dx;$$

$$10) \int_{\pi/8}^{\pi/4} \operatorname{ctg}^2 2x dx; \quad 11) \int_0^{\pi/4} (\sin^4 x + \cos^4 x) dx.$$

Rješenje.

$$1) \int_{\pi}^{2\pi} \cos \frac{x}{6} dx = 6 \sin \frac{x}{6} \Big|_{\pi}^{2\pi} = 6 \left(\sin \frac{\pi}{3} - \sin \frac{\pi}{6} \right) = 6 \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right) = 3\sqrt{3} - 3;$$

$$2) \int_0^{\pi/8} (1 + \cos 2x) dx = \left(x + \frac{1}{2} \sin 2x \right) \Big|_0^{\pi/8} = \frac{\pi}{8} + \frac{1}{2} \sin \frac{\pi}{4} = \frac{\pi}{8} + \frac{\sqrt{2}}{4};$$

$$3) \int_{\pi/3}^{\pi} \sin \left(\frac{x}{2} - \frac{\pi}{6} \right) dx = -2 \cos \left(\frac{x}{2} - \frac{\pi}{6} \right) \Big|_{\pi/3}^{\pi} = -2 \left(\cos \left(\frac{\pi}{2} - \frac{\pi}{6} \right) - \cos \left(\frac{\pi}{6} - \frac{\pi}{6} \right) \right) = -2 \left(\cos \frac{\pi}{3} - \cos 0 \right) = -2 \left(\frac{1}{2} - 1 \right) = -2 \cdot \left(-\frac{1}{2} \right) = 1;$$

$$4) \int_0^{\pi/2} \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2 dx = \int_0^{\pi/2} \left(\sin^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2} + \cos^2 \frac{x}{2} \right) dx = \int_0^{\pi/2} (1 + \sin x) dx = (x - \cos x) \Big|_0^{\pi/2} = \frac{\pi}{2} - \cos \frac{\pi}{2} + \cos 0 = \frac{\pi}{2} + 1;$$

$$5) \int_0^{\pi/2} \cos^2 \frac{x}{2} dx = \frac{1}{2} \int_0^{\pi/2} (1 + \cos x) dx = \frac{1}{2} (x + \sin x) \Big|_0^{\pi/2} = \frac{1}{2} \left(\frac{\pi}{2} + \sin \frac{\pi}{2} \right) = \frac{1}{2} \left(\frac{\pi}{2} + 1 \right);$$

$$6) \int_0^{\pi/2} \sin x \cdot \cos 3x dx = \int_0^{\pi/2} \frac{1}{2} (\sin(-2x) + \sin 4x) dx = -\frac{1}{2} \int_0^{\pi/2} \sin 2x dx + \frac{1}{2} \int_0^{\pi/2} \sin 4x dx = \left(\frac{1}{4} \cos 2x - \frac{1}{8} \cos 4x \right) \Big|_0^{\pi/2} = \frac{1}{4} \cos \pi - \frac{1}{8} \cos 2\pi - \frac{1}{4} \cos 0 + \frac{1}{8} \cos 0 = -\frac{1}{4} - \frac{1}{8} - \frac{1}{4} + \frac{1}{8} = -\frac{1}{2};$$

$$7) \int_0^{\pi/18} (\cos x \cos 2x - \sin x \sin 2x) dx = \int_0^{\pi/18} \cos 3x dx = \frac{1}{3} \sin 3x \Big|_0^{\pi/18} = \\ \frac{1}{3} \left(\sin \frac{\pi}{6} - \sin 0 \right) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6};$$

$$8) \int_{\pi/3}^{\pi/2} \cos^4 \left(x - \frac{\pi}{12} \right) dx = \int_{\pi/3}^{\pi/2} \frac{1}{8} \left(\cos \left(4x - \frac{\pi}{3} \right) + 4 \cos \left(2x - \frac{\pi}{6} \right) + 3 \right) dx = \\ \frac{1}{8} \left[\frac{1}{4} \sin \left(4x - \frac{\pi}{3} \right) + 2 \sin \left(2x - \frac{\pi}{6} \right) + 3x \right] \Big|_{\pi/3}^{\pi/2} = \frac{1}{8} \left[\frac{1}{4} \sin \frac{5\pi}{3} + 2 \sin \frac{5\pi}{6} + \right. \\ \left. \frac{3\pi}{2} - \frac{1}{4} \sin \pi - 2 \sin \frac{\pi}{2} - \pi \right] = \frac{1}{8} \left[\frac{1}{4} \cdot \left(-\frac{\sqrt{3}}{2} \right) + 2 \cdot \frac{1}{2} + \frac{3\pi}{2} - \frac{1}{4} \cdot 0 - 2 \cdot 1 - \pi \right] = \\ \frac{1}{8} \left(-\frac{\sqrt{3}}{8} + 1 - 2 + \frac{\pi}{2} \right) = \frac{1}{8} \left(\frac{\pi}{2} - 1 - \frac{\sqrt{3}}{8} \right);$$

$$9) \int_{\pi/6}^{\pi/4} (\operatorname{tg} x + \operatorname{ctg} x)^{-1} dx = \int_{\pi/6}^{\pi/4} \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)^{-1} dx = \int_{\pi/6}^{\pi/4} \left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right)^{-1} dx = \\ = \int_{\pi/6}^{\pi/4} \sin x \cos x dx = \frac{1}{2} \int_{\pi/6}^{\pi/4} \sin 2x dx = -\frac{1}{4} (\cos 2x) \Big|_{\pi/6}^{\pi/4} = -\frac{1}{4} \left(\cos \frac{\pi}{2} - \cos \frac{\pi}{3} \right) = \\ = -\frac{1}{4} \left(0 - \frac{1}{2} \right) = \frac{1}{8};$$

$$10) \int_{\pi/8}^{\pi/4} \operatorname{ctg}^2 2x dx = \int_{\pi/8}^{\pi/4} \frac{\cos^2 2x}{\sin^2 2x} dx = \int_{\pi/8}^{\pi/4} \frac{1 - \sin^2 2x}{\sin^2 2x} dx = \int_{\pi/8}^{\pi/4} \left(\frac{1}{\sin^2 2x} - \right. \\ \left. 1 \right) dx = \left(-\frac{1}{2} \operatorname{ctg} 2x - x \right) \Big|_{\pi/8}^{\pi/4} = -\frac{1}{2} \operatorname{ctg} \frac{\pi}{2} - \frac{\pi}{4} + \frac{1}{2} \operatorname{ctg} \frac{\pi}{4} + \frac{\pi}{8} = \frac{1}{2} - \frac{\pi}{8};$$

$$11) \int_0^{\pi/4} (\sin^4 x + \cos^4 x) dx = \int_0^{\pi/4} [(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x] dx = \\ = \int_0^{\pi/4} \left(1 - \frac{1}{2} \sin^2 2x \right) dx = \int_0^{\pi/4} \left(1 - \frac{1}{2} \cdot \frac{1}{2} (1 - \cos 4x) \right) dx = \int_0^{\pi/4} \left(\frac{3}{4} + \right. \\ \left. \frac{1}{4} \cos 4x \right) dx = \left(\frac{3}{4}x + \frac{1}{16} \sin 4x \right) \Big|_0^{\pi/4} = \frac{3\pi}{16} + \frac{1}{16} \sin \pi - \frac{1}{16} \sin 0 = \frac{3\pi}{16};$$