Zadatak 11. Izračunaj:

1) $\displaystyle\int_{\pi}^{3\pi/2} \frac{2dx}{\sin^2\left(\frac{x}{3}-\frac{\pi}{6}\right)\cdot\cos^2\left(\frac{x}{3}-\frac{\pi}{6}\right)}$;

2) $\displaystyle\int_{\pi/3}^{\pi/2} \frac{3dx}{\left(\cos^2\left(x-\frac{\pi}{3}\right)-\sin^2\left(x-\frac{\pi}{3}\right)\right)^2}$;

3) $\displaystyle\int_{\pi/6}^{\pi/3} \left(\cos^2\left(x+\frac{\pi}{3}\right)-\sin^2\left(x+\frac{\pi}{3}\right)\right)dx$;

4) $\displaystyle\int_{\pi/8}^{3\pi/8} 12\sin\left(\frac{\pi}{8}-x\right)\cos\left(\frac{\pi}{8}-x\right)dx$;

5) $\displaystyle\int_{\pi/12}^{\pi/24} \left(\cos^2\left(2x-\frac{\pi}{4}\right)-\sin^2\left(2x-\frac{\pi}{4}\right)\right)dx$;

6) $\displaystyle\int_{-1-\pi/12}^{1-\pi/4} \left(\sin\left(\frac{x}{2}+\frac{\pi}{12}\right)+\cos\left(\frac{x}{2}+\frac{\pi}{12}\right)\right)^2 dx$.

Rješenje.

1) $\displaystyle\int_{\pi}^{\frac{3\pi}{2}} \frac{2dx}{\sin^2\left(\frac{x}{3}-\frac{\pi}{6}\right)\cdot\cos^2\left(\frac{x}{3}-\frac{\pi}{6}\right)} = \int_{\pi}^{\frac{3\pi}{2}} \frac{8dx}{\sin^2 2\left(\frac{x}{3}-\frac{\pi}{6}\right)} = 8\left(-\operatorname{ctg}\left(\frac{2x}{3}-\frac{\pi}{3}\right)\cdot\frac{3}{2}\right)\Big|_{\pi}^{\frac{3\pi}{\pi}} = 12\left[-\operatorname{ctg}\left(\pi-\frac{\pi}{3}\right)+\operatorname{ctg}\left(\frac{2\pi}{3}-\frac{\pi}{3}\right)\right] = 12\left[-\operatorname{ctg}\frac{2\pi}{3}+\operatorname{ctg}\frac{\pi}{3}\right] =$
$12\left(\frac{\sqrt{3}}{3}+\frac{\sqrt{3}}{3}\right) = 12\cdot\frac{2\sqrt{3}}{3} = 8\sqrt{3}$;

2) $\displaystyle\int_{\pi/3}^{\pi/2} \frac{3dx}{\left(\cos^2\left(x-\frac{\pi}{3}\right)-\sin^2\left(x-\frac{\pi}{3}\right)\right)^2} = \int_{\pi/3}^{\pi/2} \frac{3dx}{\cos^2\left(2x-\frac{2\pi}{3}\right)} = 3\cdot\frac{1}{2}\cdot\operatorname{tg}\left(2x-\frac{2\pi}{3}\right)\Big|_{\pi/3}^{\pi/2} = \frac{3}{2}\left(\operatorname{tg}\frac{\pi}{3}-\operatorname{tg}0\right) = \frac{3}{2}\cdot\sqrt{3}$;

3) $\displaystyle\int_{\pi/6}^{\pi/3} \left(\cos^2\left(x+\frac{\pi}{3}\right)-\sin^2\left(x+\frac{\pi}{3}\right)\right)dx = \int_{\pi/6}^{\pi/3} \cos\left(2x+\frac{2\pi}{3}\right)dx = \frac{1}{2}\sin\left(2x+\frac{2\pi}{3}\right)\Big|_{\pi/6}^{\pi/3} = \frac{1}{2}\left(\sin\frac{4\pi}{3}-\sin\pi\right) = \frac{1}{2}\cdot\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{4}$;

4) $\displaystyle\int_{\pi/8}^{3\pi/8} 12\sin\left(\frac{\pi}{8}-x\right)\cos\left(\frac{\pi}{8}-x\right)dx = \int_{\pi/8}^{3\pi/8} 6\sin\left(\frac{\pi}{4}-2x\right)dx = 6\cdot\frac{1}{2}\cos\left(\frac{\pi}{4}-2x\right)\Big|_{\pi/8}^{3\pi/8} = 3\left(\cos\left(-\frac{\pi}{2}\right)-\cos 0\right) = -3$;

5) $\displaystyle\int_{\pi/12}^{\pi/24} \left(\cos^2\left(2x-\frac{\pi}{4}\right)-\sin^2\left(2x-\frac{\pi}{4}\right)\right)dx = \int_{\pi/12}^{\pi/24} \cos\left(4x-\frac{\pi}{2}\right)dx = \frac{1}{4}\sin\left(4x-\frac{\pi}{2}\right)\Big|_{\pi/12}^{\pi/24} = \frac{1}{4}\left(\sin\left(-\frac{\pi}{3}\right)-\sin\left(-\frac{\pi}{6}\right)\right) = \frac{1}{4}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right) =$

$\frac{1}{8}(1-\sqrt{3})$;

6) $\int_{-1-\pi/12}^{1-\pi/4}\left(\sin\left(\frac{x}{2}+\frac{\pi}{12}\right)+\cos\left(\frac{x}{2}+\frac{\pi}{12}\right)\right)^2 dx = \int_{-1-\pi/12}^{1-\pi/4}\left[\sin^2\left(\frac{x}{2}+\frac{\pi}{12}\right)+\right.$
$\left.2\sin\left(\frac{x}{2}+\frac{\pi}{12}\right)\cos\left(\frac{x}{2}+\frac{\pi}{12}\right)+\cos^2\left(\frac{x}{2}+\frac{\pi}{12}\right)\right]dx = \int_{-1-\pi/12}^{1-\pi/4}\left[1+\sin\left(x+\frac{\pi}{6}\right)\right]dx$
$=\left[x-\cos\left(x+\frac{\pi}{6}\right)\right]\Big|_{-1-\pi/12}^{1-\pi/4} = 1-\frac{\pi}{4}-\cos\left(1-\frac{\pi}{4}+\frac{\pi}{6}\right)+1+\frac{\pi}{12}+\cos\left(-1-\frac{\pi}{12}+\frac{\pi}{6}\right)$
$=2-\frac{\pi}{6}-\cos\left(1-\frac{\pi}{12}\right)+\cos\left(-1+\frac{\pi}{12}\right) = 2-\frac{\pi}{6}-\cos\left(1-\frac{\pi}{12}\right)+\cos\left(1-\right.$
$\left.\frac{\pi}{12}\right)=2-\frac{\pi}{6}$.