Rješenja zadataka 5.4

Zadatak 1. Svođenjem pod znak diferencijala ili metodom supstitucije izračunaj integrale:

1) $\int \sqrt{3+x}\,dx$; **2)** $\int \sqrt{4-3x}\,dx$;

3) $\int \sqrt{1-4x}\,dx$; **4)** $\int \sqrt{3x+2}\,dx$;

5) $\int \sqrt[3]{4-5x}\,dx$; **6)** $\int \sqrt[4]{2+3x}\,dx$;

7) $\int \sqrt[4]{(x-2)^3}\,dx$; **8)** $\int \frac{dx}{\sqrt{3-4x}}$;

9) $\int \frac{dx}{\sqrt[4]{(2-3x)^3}}$; **10)** $\int \sqrt[3]{4x+3}\,dx$;

11) $\int \frac{dx}{\sqrt[3]{(3-2x)^4}}$; **12)** $\int \sqrt[3]{(2x+1)^5}\,dx$.

Rješenje. **1)** $\int \sqrt{3+x}\,dx = \left\{ \begin{array}{c} 3+x=t \\ dx=dt \end{array} \right\} = \int t^{\frac{1}{2}}dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}(x+3)\sqrt{x+3}+C$

ili $\int \sqrt{3+x}\,dx = \int \sqrt{3+x}\,d(3+x) = \frac{2}{3}(3+x)^{\frac{3}{2}} + C = \frac{2}{3}\sqrt{(3+x)^3}+C$;

2) $\int \sqrt{4-3x}\,dx = \left\{ \begin{array}{c} 4-3x=t \\ -3dx=dt \\ dx=-\frac{1}{3}dt \end{array} \right\} = \int \sqrt{t}\left(-\frac{1}{3}\right)dt = -\frac{1}{3}\cdot\frac{2}{3}\sqrt{t^3}+C =$
$-\frac{2}{9}\sqrt{(4-3x)^3}+C$

ili $\int \sqrt{4-3x}\,dx = \left\{ \begin{array}{c} 4-3x=t^2 \\ -3dx=2tdt \\ dx=-\frac{2}{3}tdt \end{array} \right\} = \int t\left(-\frac{2}{3}\right)tdt = -\frac{2}{3}\int t^2dt =$
$-\frac{2}{3}\cdot\frac{t^3}{3}+C = -\frac{2}{9}\sqrt{(4-3x)^3}+C$;

3) $\int \sqrt{1-4x}\,dx = \left\{ \begin{array}{c} 1-4x=t^2 \\ -4dx=2tdt \\ dx=-\frac{1}{2}tdt \end{array} \right\} = \int t\left(-\frac{1}{2}t\right)dt = -\frac{1}{2}\int t^2dt =$
$-\frac{1}{2}\frac{t^3}{3}+C = -\frac{1}{6}\sqrt{(1-4x)^3}+C$;

4) $\int \sqrt{3x+2}\,dx = \left\{ \begin{array}{c} 3x+2=t^2 \\ 3dx=2tdt \\ dx=\frac{2}{3}tdt \end{array} \right\} = \int \frac{2}{3}t^2dt = \frac{2}{9}t^3+C = \frac{2}{9}\sqrt{(3x+2)^3}+$
C;

5) $\int \sqrt[3]{4-5x}\,dx = \left\{\begin{array}{c} 4-5x=t^3 \\ -5dx = 3t^2dt \\ dx = -\frac{3}{5}t^2dt \end{array}\right\} = \int\left(-\frac{3}{5}t^3\right)dt = -\frac{3}{20}t^4 + C =$
$-\frac{3}{20}\sqrt[3]{(4-5x)^4} + C;$

6) $\int \sqrt[4]{2+3x}\,dx = \left\{\begin{array}{c} 2+3x=t^4 \\ 3dx = 4t^3dt \\ dx = \frac{4}{3}t^3dt \end{array}\right\} = \int \frac{4}{3}t^4dt = \frac{4}{15}t^5 + C =$
$\frac{4}{15}\sqrt[4]{(2+3x)^5} + C;$

7) $\int \sqrt[4]{(x-2)^3}\,dx = \left\{\begin{array}{c} x-2=t^4 \\ dx = 4t^3dt \end{array}\right\} = \int 4t^6dt = \frac{4}{7}t^7 + C = \frac{4}{7}\sqrt[4]{(x-2)^7} +$
$C;$

8) $\int \frac{dx}{\sqrt{3-4x}} = \left\{\begin{array}{c} 3-4x=t^2 \\ -4dx = 2tdt \\ dx = -\frac{1}{2}tdt \end{array}\right\} = \int\left(-\frac{1}{2}t\cdot t^{-1}\right)dt =$
$-\frac{1}{2}t + C = -\frac{1}{2}\sqrt{3-4x} + C;$

9) $\int \frac{dx}{\sqrt[4]{(2-3x)^3}} = \int (2-3x)^{-\frac{3}{4}}\,dx = \left\{\begin{array}{c} 2-3x=t^4 \\ -3dx = 4t^3dt \\ dx = -\frac{4}{3}t^3dt \end{array}\right\} = \int t^{-3}\left(-\frac{4}{3}\right)t^3dt =$
$-\frac{4}{3}\int dt = -\frac{4}{3}t + C = -\frac{4}{3}\sqrt[4]{2-3x} + C;$

10) $\int \sqrt[3]{4x+3}\,dx = \left\{\begin{array}{c} 4x+3=t^3 \\ 4dx = 3t^2dt \\ dx = \frac{3}{4}t^2dt \end{array}\right\} = \int t\cdot\frac{3}{4}t^2dt = \frac{3}{4}\int t^3dt = \frac{3}{4}\cdot\frac{1}{4}t^4 +$
$C = \frac{3}{16}\sqrt[3]{(4x+3)^4} + C;$

11) $\int \frac{dx}{\sqrt[3]{(3-2x)^4}} = \int (3-2x)^{-\frac{4}{3}}dx = \left\{\begin{array}{c} 3-2x=t^3 \\ -2dx = 3t^2dt \\ dx = -\frac{3}{2}t^2dt \end{array}\right\} = -\frac{3}{2}\int t^{-4+2}dt =$
$-\frac{3}{2}\int t^{-2}dt = -\frac{3}{2}(-1)\frac{1}{t} + C = \frac{3}{2\sqrt[3]{3-2x}} + C;$

12) $\int \sqrt[3]{(2x+1)^5}\,dx = \left\{\begin{array}{c} 2x+1=t^3 \\ 2dx = 3t^2dt \\ dx = \frac{3}{2}t^2dt \end{array}\right\} = \int \frac{3}{2}t^7dt = \frac{3}{16}t^8 + C =$
$\frac{3}{16}\sqrt[3]{(2x+1)^8} + C.$