Zadatak 2. Svođenjem pod znak diferencijala ili metodom supstitucije izračunaj integrale:

1) $\int (1+x)^3\,dx$; 2) $\int \frac{x\,dx}{1+x^2}$;

3) $\int \frac{x^2\,dx}{x^3+1}$; 4) $\int \frac{x^2\,dx}{(x^3+1)^2}$;

5) $\int \frac{x\,dx}{2+x^2}$; 6) $\int \frac{x\,dx}{\sqrt{1+x^2}}$;

7) $\int \frac{x^2\,dx}{\sqrt[3]{5+x^3}}$; 8) $\int \frac{x\,dx}{\sqrt{1-x^2}}$;

9) $\int \frac{x^2\,dx}{(x^3+1)^2}$; 10) $\int \frac{2x+1}{x^2+x-3}\,dx$;

11) $\int \frac{dx}{x\sqrt{1-x}}$; 12) $\int \frac{4x+3}{\sqrt{2x^2+3x+1}}dx$.

Rješenje.

1) $\int (1+x)^3dx = \int (1+x)^3 d(1+x) = \frac{1}{4}(1+x)^4 + C$;

2) $\int \frac{x\,dx}{1+x^2} = \frac{1}{2}\int \frac{2x\,dx}{1+x^2} = \frac{1}{2}\int \frac{d(1+x^2)}{1+x^2} = \frac{1}{2}\ln(1+x^2)+C$;

3) $\int \frac{x^2dx}{x^3+1} = \frac{1}{3}\cdot\int \frac{3x^2dx}{x^3+1} = \frac{1}{3}\int \frac{d(x^3+1)}{x^3+1} = \frac{1}{3}\ln|x^3+1|+C$;

4) $\int \frac{x^2dx}{(x^3+1)^2} = \frac{1}{3}\int \frac{3x^2dx}{(x^3+1)^2} = \int \frac{d(x^3+1)}{(x^3+1)^2} = -\frac{1}{3(x^3+1)}+C$;

5) $\int \frac{x\,dx}{2+x^2} = \frac{1}{2}\int \frac{2x\,dx}{2+x^2} = \frac{1}{2}\int \frac{d(2+x^2)}{2+x^2} = \frac{1}{2}\ln(2+x^2)+C$;

6) $\int \frac{x\,dx}{\sqrt{1+x^2}} = \left\{\begin{matrix} 1+x^2=t^2 \\ 2x\,dx = 2tdt \\ x\,dx = tdt \end{matrix}\right\} = \int \frac{tdt}{t} = t+C = \sqrt{1+x^2}+C$;

7) $\int \frac{x^2dx}{\sqrt[3]{5+x^3}} = \left\{\begin{matrix} 5+x^3=t^3 \\ 3x^2dx = 3t^2dt \\ x^2dx = t^2dt \end{matrix}\right\} = \int \frac{t^2dt}{t} = \frac{1}{2}t^2+C = \frac{1}{2}\sqrt[3]{(5+x^3)^2}+C$;

8) $\int \frac{x\,dx}{\sqrt{1-x^2}} = \left\{\begin{matrix} 1-x^2=t^2 \\ -2x\,dx = 2tdt \\ x\,dx = -tdt \end{matrix}\right\} = \int \frac{-tdt}{t} = -t+C = -\sqrt{1-x^2}+C$;

9) $\int \frac{x^2dx}{\sqrt{x^3+1}} = \left\{\begin{matrix} x^3+1=t^2 \\ 3x^2dx = 2tdt \\ x^2dx = \frac{2}{3}tdt \end{matrix}\right\} = \int \frac{2}{3}\cdot\frac{tdt}{t} = \frac{2}{3}t+C = \frac{2}{3}\sqrt{x^3+1}+C$;

10) $\int \frac{2x+1}{x^2+x-3}dx = \left\{\begin{matrix} x^2+x-3=t \\ (2x+1)dx = dt \end{matrix}\right\} = \int \frac{dt}{t} = \ln|t|+C = \ln|x^2+x-3|+C$;

11) $\int \frac{dx}{x\sqrt{1-x}} = \left\{\begin{matrix} 1-x=t^2 \\ -dx=2tdt \end{matrix}\right\} = \int \frac{-2tdt}{(1-t^2)t} = -2\int \frac{dt}{1-t^2} = -2\cdot\frac{1}{2}\ln\left|\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right|+C$
$= \ln\left|\frac{1-\sqrt{1-x}}{1+\sqrt{1-x}}\right|+C$;

12) $\int \frac{4x+3}{\sqrt{2x^2+3x+1}}\,dx = \left\{ \begin{array}{l} 2x^2+3x+1=t^2 \\ (4x+3)dx=2tdt \end{array} \right\} = \int \frac{2tdt}{t} = 2t + C =$
$2\sqrt{2x^2+3x-1} + C\,.$