Zadatak 3. Izračunaj integrale:

1) $\int \frac{\ln x}{x}\,dx$;

2) $\int \frac{dx}{x\ln x}$;

3) $\int \frac{dx}{x\ln^2 x}$;

4) $\int \frac{\ln^3 x}{x}\,dx$;

5) $\int \frac{\sqrt{1+\ln x}}{x}\,dx$;

6) $\int \frac{e^x}{e^x+2}\,dx$;

7) $\int xe^{x^2}\,dx$;

8) $\int \frac{e^x\,dx}{e^x+1}$;

9) $\int e^{-x^2}x\,dx$;

10) $\int x\,e^{2x^2-1}\,dx$;

11) $\int \frac{e^{2x}\,dx}{1-3e^{2x}}$;

12) $\int (3x-1)e^{3x^2-2x+1}dx$.

Rješenje.

1) $\int \frac{\ln x}{x}\,dx = \int \ln x d(\ln x) = \frac{1}{2}\ln^2|x| + C$;

2) $\int \frac{dx}{x\ln x} = \int \frac{d(\ln x)}{\ln x} = \ln|\ln|x|| + C$;

3) $\int \frac{dx}{x\ln^2 x} = \int \frac{d(\ln x)}{\ln^2 x} = -\frac{1}{\ln|x|} + C$;

4) $\int \frac{\ln^3 x}{x}\,dx = \int \ln^3 x d(\ln x) = \frac{1}{4}\ln^4|x| + C$;

5) $\int \frac{\sqrt{1+\ln x}}{x}\,dx = \int \sqrt{1+\ln x}\,d(1+\ln x) = \frac{2}{3}\sqrt{(1+\ln|x|)^3} + C$;

6) $\int \frac{e^x}{e^x+2}\,dx = \int \frac{d(e^x+2)}{e^x+2} = \ln(e^x+2) + C$;

7) $\int xe^{x^2}\,dx = \frac{1}{2}\int e^{x^2}d(x^2) = \frac{1}{2}e^{x^2} + C$;

8) $\int \frac{e^x\,dx}{\sqrt{e^x+1}} = \left\{\begin{array}{l} e^x+1=t^2 \\ e^x dx = 2tdt \end{array}\right\} = \int \frac{2tdt}{t} = 2t + C = 2\sqrt{e^x+1} + C$;

9) $\int e^{-x^2}x\,dx = -\frac{1}{2}\int e^{-x^2}d(-x^2) = -\frac{1}{2}e^{-x^2} + C$;

10) $\int x\,e^{2x^2-1}\,dx = \frac{1}{4}\int e^{2x^2-1}d(2x^2-1) = \frac{1}{4}e^{2x^2-1} + C$;

11) $\int \frac{e^{2x}\,dx}{1-3e^{2x}} = -\frac{1}{6}\int \frac{d(1-3e^{2x})}{1-3e^{2x}} = -\frac{1}{6}\ln|1-3e^{2x}| + C$;

12) $\int (3x-1)e^{3x^2-2x+1}dx = \frac{1}{2}\int e^{3x^2-2x+1}d(3x^2-2x+1) = \frac{1}{2}e^{3x^2-2x+1} + C$.