Zadatak 4. Uvođenjem nove varijable izračunaj:

1) $\int 4\sin 3x\,dx$;

2) $\int (\sin 2x - \cos 5x)\,dx$;

3) $\int \sin x\cos^3 x\,dx$;

4) $\int \sin^2 x\cos x\,dx$;

5) $\int \frac{\sin x}{\cos^3 x}\,dx$;

6) $\int \frac{\cos x}{\sin^4 x}\,dx$;

7) $\int \frac{\sin x}{1+3\cos x}\,dx$;

8) $\int \frac{\cos x}{1+2\sin x}\,dx$;

9) $\int \operatorname{ctg} x\,dx$;

10) $\int \frac{\cos 2x}{\sin x\,\cos x}\,dx$;

11) $\int \frac{\sin x}{\sqrt{1+2\cos x}}\,dx$;

12) $\int \sin^2 x\,dx$.

Rješenje.

1) $\int 4\sin 3x\,dx = \left\{\begin{array}{c} 3x = t \\ 3dx = dt \end{array}\right\} = \frac{4}{3}\int \sin t dt = -\frac{4}{3}\cos t + C = -\frac{4}{3}\cos 3x + C$;

2) $\int (\sin 2x - \cos 5x)\,dx = \int \sin 2x dx - \int \cos 5x dx = \left\{\begin{array}{c} 2x = t \\ 2dx = dt \end{array}\right\} = \frac{1}{2}\int \sin t\,dt - \int \cos 5x\,dx = \left\{\begin{array}{c} 5x = z \\ 5dx = dz \end{array}\right\} = -\frac{1}{2}\cos t - \frac{1}{5}\int \cos z\,dz = -\frac{1}{2}\cos 2x - \frac{1}{5}\sin 5x + C$;

3) $\int \sin x\cos^3 x\,dx = \left\{\begin{array}{c} \cos x = t \\ -\sin x dx = dt \end{array}\right\} = -\int t^3 dt = -\frac{1}{4}t^4 + C = -\frac{1}{4}\cos^4 x + C$;

4) $\int \sin^2 x\cos x\,dx = \left\{\begin{array}{c} t = sinx \\ dt = cosx\,dx \end{array}\right\} = \int t^2\,dt = \frac{1}{3}t^3 + C == \frac{1}{3}\sin^3 x + C$;

5) $\int \frac{\sin x}{\cos^3 x}\,dx = \left\{\begin{array}{c} t = \cos x \\ dt = -\sin x\,dx \end{array}\right\} = -\int \frac{dt}{t^3} = -\left(-\frac{1}{2}\right)\cdot\frac{1}{t^2} + C = \frac{1}{2\cos^2 x} + C$;

6) $\int \frac{\cos x}{\sin^4 x}\,dx = \left\{\begin{array}{c} t = \sin x \\ dt = \cos x\,dx \end{array}\right\} = \int \frac{dt}{t^4} = -\frac{1}{3}\cdot\frac{1}{t^3} + C = -\frac{1}{3\sin^3 x} + C$;

7) $\int \frac{\sin x}{1+3\cos x}\,dx = \left\{\begin{array}{c} 1+3\cos x = t \\ -3\sin x dx = dt \end{array}\right\} = -\frac{1}{3}\int \frac{dt}{t} = -\frac{1}{3}\ln|t| + C = -\frac{1}{3}\ln|1+3\cos x| + C$;

8) $\int \frac{\cos x}{1+2\sin x}\,dx = \left\{\begin{array}{c} 1+2\sin x = t \\ 2\cos x\,dx = dt \end{array}\right\} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| + C = \frac{1}{2}\ln|1+2\sin x| + C$;

9) $\int \operatorname{ctg} x\,dx = \int \frac{\cos x}{\sin x}dx = \left\{ \begin{array}{c} t = \sin x \\ dt = \cos x\,dx \end{array} \right\} = \int \frac{dt}{t} = \ln|t| + C =$
$\ln|\sin x| + C$;

10) $\int \frac{\cos 2x}{\sin x\,\cos x}\,dx = \int \frac{2\cos 2x}{\sin 2x}dx = \int \frac{d(\sin 2x)}{\sin 2x} = \ln|\sin 2x| + C$;

11) $\int \frac{\sin x}{\sqrt{1+2\cos x}}\,dx = \left\{ \begin{array}{c} 1+2\cos x = t^2 \\ -2\sin x dx = 2tdt \\ \sin x dx = -tdt \end{array} \right\} = \int \frac{-tdt}{t} = -t + C =$
$-\sqrt{1+2\cos x} + C$;

12) $\int \sin^2 x\,dx = \frac{1}{2}\int (1-\cos 2x)dx = \left\{ \begin{array}{c} t = 2x \\ dt = 2dx \end{array} \right\} = \frac{1}{2}x - \frac{1}{4}\int \cos t\,dt =$
$\frac{1}{2}x - \frac{1}{4}\sin t = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$.