

Zadatak 5. Izračunaj integrale:

$$1) \int_0^{\pi/2} \cos^2 \frac{x}{2} dx; \quad 2) \int_0^{\pi/4} (1 + \cos 2x) dx;$$

$$3) \int_{-1}^1 (1 + 2x)^4 dx; \quad 4) \int_0^1 \frac{x}{(x+1)^2} dx.$$

Rješenje.

$$1) \int_0^{\pi/2} \cos^2 \frac{x}{2} dx = \frac{1}{2} \int_0^{\pi/2} (1 + \cos x) dx = \frac{1}{2} (x + \sin x) \Big|_0^{\pi/2} = \frac{1}{2} \left(\frac{\pi}{2} + 1 \right);$$

$$2) \int_0^{\pi/4} (1 + \cos 2x) dx = \left(x + \frac{1}{2} \sin 2x \right) \Big|_0^{\pi/4} = \frac{\pi}{4} + \frac{1}{2} \cdot 1 = \frac{1}{2} \left(\frac{\pi}{2} + 1 \right);$$

$$3) \int_{-1}^1 (1 + 2x)^4 dx = \frac{1}{2} \int_{-1}^1 2(1 + 2x)^4 dx = \frac{1}{2} \int_{-1}^1 (1 + 2x)^4 d(1 + 2x) = \frac{1}{10} (1 + 2x)^5 \Big|_{-1}^1 = \frac{1}{10} (243 + 1) = \frac{244}{10} = \frac{122}{5};$$

$$4) \int_0^1 \frac{x}{(x+1)^2} dx = \int_0^1 \frac{x+1-1}{(x+1)^2} dx = \int_0^1 \frac{dx}{x+1} - \int \frac{d(x+1)}{(x+1)^2} = \left(\ln|x+1| + \frac{1}{x+1} \right) \Big|_0^1 = \ln 2 + \frac{1}{2} - \ln 1 - 1 = \ln 2 - \frac{1}{2}.$$