Zadatak 6. Izračunaj:

1) $\displaystyle\int_0^3 \frac{2x}{x^2+1}\,dx\,;$ **2)** $\displaystyle\int_2^3 \frac{3x^2}{x^3-1}\,dx\,;$

3) $\displaystyle\int_0^6 (x-3)e^{x^2-6x}\,dx\,;$

4) $\displaystyle\int_0^{\frac{\pi}{2}} \sin x\cdot e^{\cos x}\,dx;$ **5)** $\displaystyle\int_0^1 \frac{x\,dx}{(x+1)^2}\,;$

6) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin 2x\cdot e^{\cos^2 x}\,dx\,.$

Rješenje.

1) $\displaystyle\int_0^3 \frac{2x}{x^2+1}\,dx = \int_0^3 \frac{d(x^2+1)}{x^2+1} = \ln(x^2+1)\Big|_0^3 = \ln 10 - \ln 1 = \ln 10\,;$

2) $\displaystyle\int_2^3 \frac{3x^2}{x^3-1}\,dx = \int_2^3 \frac{d(x^3-1)}{x^3-1} = \ln|x^3-1|\Big|_2^3 = \ln 26 - \ln 7 = \ln\frac{26}{7}\,;$

3) $\displaystyle\int_0^6 (x-3)e^{x^2-6x}\,dx = \frac{1}{2}\int_0^6 e^{x^2-6x}d(x^2-6x) = \frac{1}{2}e^{x^2-6x}\Big|_0^6 = \frac{1}{2}(e^0-e^0) = 0\,;$

4) $\displaystyle\int_0^{\pi/2} \sin x\cdot e^{\cos x}\,dx = -\int_0^{\pi/2} e^{\cos x}d(\cos x) = -e^{\cos x}\Big|_0^{\pi/2} = -e^0+e^1 = e-1\,;$

5) $\displaystyle\int_0^b \frac{ax\,dx}{ax^2+b} = \frac{1}{2}\int_0^b \frac{2axdx}{ax^2+b} = \frac{1}{2}\int_0^b \frac{d(ax^2+b)}{ax^2+b} = \frac{1}{2}\ln|ax^2+b|\Big|_0^b = \frac{1}{2}(\ln|ab^2+b| - \ln|b|) = \frac{1}{2}\ln\left|\frac{b(ab+1)}{b}\right| = \frac{1}{2}\ln|ab+1|\,;$

6) $\displaystyle\int_{-\pi/4}^{\pi/4} \sin 2xe^{\cos^2 x}\,dx - \int_{-\pi/4}^{\pi/4} e^{\cos^2 x}d(\cos^2 x) = -e^{\cos^2 x}\Big|_{-\pi/4}^{\pi/4} = -(e^{\frac{1}{2}} - e^{\frac{1}{2}}) = 0\,.$