Rješenja zadataka 5.5

Zadatak 1. Metodom parcijalne integracije odredi sljedeće integrale:

1) $\int x \sin x\,dx$; **2)** $\int x e^x\,dx$;

3) $\int x \ln x\,dx$; **4)** $\int x^2 \ln x\,dx$;

5) $\int x\sqrt{1+x}\,dx$; **6)** $\int e^x \cos x\,dx$;

7) $\int x^2 \sin x\,dx$; **8)** $\int x^2 e^x\,dx$;

9) $\int x^3 e^{x^2}\,dx$.

Rješenje. **1)** $\int x \sin x\,dx = \left\{ \begin{matrix} x = u & \sin dx = dv \\ dx = du & v = -\cos x \end{matrix} \right\} = -x\cos x + \int \cos x dx =$
$-x\cos x + \sin x + C$;

2) $\int x e^x\,dx = \left\{ \begin{matrix} x = u & dx = du \\ e^x dx = dv & e^x = v \end{matrix} \right\} = xe^x - \int e^x dx = xe^x - e^x + C$;

3) $\int x \ln x\,dx = \left\{ \begin{matrix} \ln x = u & x dx = dv \\ \dfrac{dx}{x} = du & \dfrac{x^2}{2} = v \end{matrix} \right\} = \dfrac{x^2}{2}\ln x - \int \dfrac{x^2}{2} \cdot \dfrac{dx}{x} =$
$\dfrac{x^2}{2}\ln|x| - \dfrac{1}{2}\cdot\dfrac{x^2}{2} + C = \dfrac{x^2}{2}\ln|x| - \dfrac{x^2}{4} + C$;

4) $\int x^2 \ln x\,dx = \left\{ \begin{matrix} \ln x = u & x^2 dx = dv \\ \dfrac{dx}{x} = du & \dfrac{x^3}{3} = v \end{matrix} \right\} = \dfrac{x^3}{3}\ln|x| - \int \dfrac{x^3}{3} \cdot \dfrac{dx}{x} =$
$\dfrac{x^3}{3}\ln|x| - \dfrac{1}{9}x^3 + C$;

5) $\int x\sqrt{1+x}\,dx = \left\{ \begin{matrix} x = u & \dfrac{3}{2}\sqrt{(1+x)^3} = v \\ dx = du & \sqrt{1+x}dx = dv \end{matrix} \right\}$
$= \dfrac{2}{3}x\sqrt{(1+x)^3} - \dfrac{2}{3}\int \sqrt{(1+x)^3}dx = \dfrac{2}{3}x\sqrt{(1+x)^3} - \dfrac{4}{15}\sqrt{(1+x)^5} + C$;

6) $\int e^x \cos x\,dx = \left\{ \begin{matrix} e^x dx = dv & \cos x = u \\ e^x = v & -\sin x dx = du \end{matrix} \right\} = e^x \cos x + \int e^x \sin x dx =$
$\left\{ \begin{matrix} e^x dx = dv & \sin x = u \\ e^x = v & \cos x dx = du \end{matrix} \right\} = e^x \cos x + e^x \sin x - \int e^x \cos x dx \implies e^x \cos x dx =$
$\dfrac{1}{2}e^x(\sin x + \cos x) + C$;

7) $\int x^2 \sin x\,dx = \left\{ \begin{matrix} x^2 = u & \sin x dx = dv \\ 2x dx = du & -\cos x = v \end{matrix} \right\} = -x^2\cos x + 2\int x\cos x dx =$
$\left\{ \begin{matrix} x = u & \cos x dx = dv \\ dx = du & \sin x = v \end{matrix} \right\} = -x^2\cos x + 2x\sin x - 2\int \sin x dx = -x^2\cos x +$
$2x\sin x + 2\cos x + C$;

8) $\int x^2 e^x\,dx = \left\{ \begin{matrix} x^2 = u & e^x dx = dv \\ 2xdx = du & e^x = v \end{matrix} \right\} = x^2e^x - 2\int xe^x dx$

$= \left\{ \begin{matrix} x = u & e^x dx = dv \\ dx = du & e^x = v \end{matrix} \right\} = x^2e^x - 2xe^x + 2\int e^x dx = x^2e^x - 2xe^x + 2e^x + C;$

9) $\int x^3 e^{x^2}\,dx = \frac{1}{2}\int x^2 2xe^{x^2} dx = \left\{ \begin{matrix} x^2 = u & 2xe^{x^2} dx = dv \\ 2xdx = du & e^{x^2} = v \end{matrix} \right\}$

$= \frac{1}{2}x^2e^{x^2} - \frac{1}{2}\int 2xe^{x^2} dx = \frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^{x^2} + C.$