Zadatak 2. Metodom parcijalne integracije odredi sljedeće integrale:

1) $\int x\,e^{2x}\,dx$;
2) $\int x^3 \ln x\,dx$;
3) $\int e^x \sin x\,dx$;
4) $\int \ln x\,dx$;
5) $\int x \ln(x+1)\,dx$;
6) $\int x^2 \cos x\,dx$;
7) $\int \ln^2 x\,dx$;
8) $\int \frac{\ln x}{x^2}\,dx$;
9) $\int e^{\sqrt{x}}\,dx$.

Rješenje.

1) $\int x\,e^{2x}\,dx = \left\{\begin{array}{ll} x=u & e^{2x}dx=dv \\ dx=du & \frac{1}{2}e^{2x}=v \end{array}\right\} = \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C = \frac{1}{4}e^{2x}(2x-1) + C$;

2) $\int x^3 \ln x\,dx = \left\{\begin{array}{ll} \ln x=u & x^3dx=dv \\ \frac{dx}{x}=du & \frac{x^4}{4}=v \end{array}\right\} = \frac{1}{4}x^4\ln x - \frac{1}{4}\int x^3dx = \frac{1}{4}x^4\ln|x| - \frac{1}{4}\cdot\frac{1}{4}x^4 + C = \frac{1}{16}x^4(4\ln|x|-1) + C$;

3) $\int e^x \sin x\,dx = \left\{\begin{array}{ll} \sin x=u & e^xdx=dv \\ \cos xdx=du & e^x=v \end{array}\right\} = e^x\sin x - \int e^x\cos xdx = \left\{\begin{array}{ll} \cos x=u & e^xdx=dv \\ -\sin xdx=du & e^x=v \end{array}\right\} = e^x\sin x - e^x\cos x - \int e^x\sin xdx \implies \int e^x\sin xdx = \frac{1}{2}e^x(\sin x - \cos x) + C$;

4) $\int \ln x\,dx = \left\{\begin{array}{ll} \ln x=u & dx=dv \\ \frac{dx}{x}=du & x=v \end{array}\right\} = x\ln x - \int dx = x\ln x - x + C = x(\ln x - 1) + C$;

5) $\int x\ln(x+1)\,dx = \left\{\begin{array}{ll} \ln(x+1)=u & xdx=dv \\ \frac{dx}{x+1}=du & \frac{x^2}{2}=v \end{array}\right\} = \frac{x^2}{2}\ln|x+1| - \frac{1}{2}\int\frac{x^2}{x+1}dx = \frac{x^2}{2}\ln|x+1| - \frac{1}{2}\int\frac{x^2-1+1}{x+1}dx = \frac{x^2}{2}\ln|x+1| - \frac{1}{2}\int(x-1)dx - \frac{1}{2}\int\frac{dx}{x+1} = \frac{x^2}{2}\ln|x+1| - \frac{1}{2}\cdot\frac{x^2}{2} + \frac{1}{2}x - \frac{1}{2}\ln|x+1| + C = \frac{x^2-1}{2}\ln|x+1| - \frac{x^2}{4} + \frac{1}{2}x + C$;

6) $\int x^2\cos x\,dx = \left\{\begin{array}{ll} x^2=u & \cos xdx=dv \\ 2xdx=du & \sin x=v \end{array}\right\} = x^2\sin x - 2\int x\sin xdx = \left\{\begin{array}{ll} x=u & \sin xdx=dv \\ dx=du & -\cos x=v \end{array}\right\} = x^2\sin x + 2x\cos x - 2\int\cos xdx = x^2\sin x + 2x\cos x - 2\sin x + C$;

7) $\int \ln^2 x\,dx = \left\{\begin{array}{ll} \ln^2 x=u & dx=dv \\ \frac{2\ln x}{x}dx=du & x=v \end{array}\right\} = x\ln^2 x - \int\frac{2\ln x}{x}\cdot xdx =$

$$x\ln^2 x - 2\int \ln x dx = \left\{ \begin{matrix} \ln x = u & dx = dv \\ \frac{dx}{x} = du & x = v \end{matrix} \right\} = x\ln^2 x - 2(x\ln x - \int dx) =$$
$$x\ln^2 x - 2x\ln x + 2x + C;$$

8) $$\int \frac{\ln x}{x^2} dx = \left\{ \begin{matrix} \ln x = u & \frac{dx}{x^2} = dv \\ \frac{dx}{x} = du & -\frac{1}{x} = v \end{matrix} \right\} = -\frac{\ln|x|}{x} + \int \frac{dx}{x^2} = -\frac{\ln|x|}{x} -$$
$$\frac{1}{x} + C = -\frac{1}{x}(1 + \ln|x|) + C;$$

9) $$\int e^{\sqrt{x}} dx = \left\{ \begin{matrix} x = t^2 \\ dx = 2tdt \end{matrix} \right\} = 2\int te^t dt = \left\{ \begin{matrix} t = u & e^t dt = dv \\ dt = du & e^t = v \end{matrix} \right\} =$$
$$2te^t - 2\int e^t dt = 2te^t - 2e^t + C = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C = 2e^{\sqrt{x}}(\sqrt{x} - 1) + C.$$