Zadatak 4. Izračunaj integrale:

1) $\int_2^3 \ln x\,dx$; **2)** $\int_2^3 xe^x dx$;

3) $\int_0^{\pi/2} x\sin x\,dx$; **4)** $\int_0^{\pi/2} x\cos x\,dx$;

5) $\int_0^{\pi/2} x\sin 2x\,dx$; **6)** $\int_0^1 xe^{-2x}\,dx$.

Rješenje. **1)** $\int_2^3 \ln x\,dx = \left\{\begin{matrix} \ln x = u & dx = dv \\ \frac{dx}{x} = du & x = v \end{matrix}\right\} = x\ln x\Big|_2^3 - \int_2^3 dx = 3\ln 3 - 2\ln 2 - x\Big|_2^3 = 3\ln 3 - 2\ln 2 - 1 = \ln 27 - \ln 4 - \ln e = \ln\frac{27}{4e}$;

2) $\int_2^3 xe^x dx = \left\{\begin{matrix} x = u & e^x dx = dv \\ dx = du & e^x = v \end{matrix}\right\} = xe^x\Big|_2^3 - \int_2^3 e^x dx = 3e^3 - 2e^2 - e^3 + e^2 = 2e^3 - e^2 = e^2(2e-1)$;

3) $\int_0^{\pi/2} x\sin x\,dx = \left\{\begin{matrix} x = u & \sin xdx = dv \\ dx = du & -\cos x = v \end{matrix}\right\} = -x\cos x\Big|_0^{\pi/2} + \int_0^{\pi/2} \cos x\,dx = -\frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2} - \sin 0 = 1$;

4) $\int_0^{\pi/2} x\cos x\,dx = \left\{\begin{matrix} x = u & \cos xdx = dv \\ dx = du & \sin x = v \end{matrix}\right\} = x\sin x\Big|_0^{\pi/2} - \int_0^{\pi/2} \sin xdx = \frac{\pi}{2}\sin\frac{\pi}{2} + \cos x\Big|_0^{\pi/2} = \frac{\pi}{2} + \cos\frac{\pi}{2} - \cos 0 = \frac{\pi}{2} - 1$;

5) $\int_0^{\pi/2} x\sin 2x\,dx = \left\{\begin{matrix} x=u & \sin 2x dx = dv \\ dx = du & -\frac{1}{2}\cos 2x = v\end{matrix}\right\} = -\frac{1}{2}x\cos 2x\Big|_0^{\pi/2} +$
$\frac{1}{2}\int_0^{\pi/2}\cos 2x dx = -\frac{\pi}{4}\cos\pi + \frac{1}{4}\sin 2x\Big|_0^{\pi/2} = \frac{\pi}{4} + \frac{1}{4}(\sin\pi - \sin 0) = \frac{\pi}{4};$

6) $\int_0^1 xe^{-2x}\,dx = \left\{\begin{matrix} x=u & e^{-2x}dx = dv \\ dx = du & -\frac{1}{2}e^{-2x} = v\end{matrix}\right\} = -\frac{1}{2}xe^{-2x}\Big|_0^1 + \frac{1}{2}\int e^{-2x}dx$
$= -\frac{1}{2}e^{-2} - \frac{1}{4}e^{-2x}\Big|_0^1 = -\frac{1}{2e^2} - \frac{1}{4}(e^{-2} - e^0) = -\frac{2}{4e^2} - \frac{1}{4e^2} + \frac{1}{4} = \frac{1}{4}\left(1 - \frac{3}{e^2}\right).$