Zadatak 3. Izračunaj površinu lika omeđenog sljedećim krivuljama:

1) $y = x^2 + 4x$, $y = x + 4$;

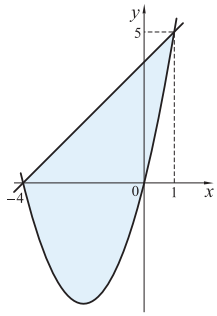
2) $y = e^x$, $y = e^{-x}$, $x = 2$;

3) $y = \dfrac{1}{1+x^2}$, $x = 0$, $x = 1$, $y = 0$;

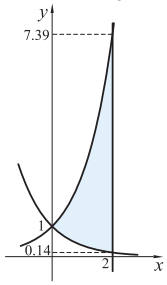
4) $y = \dfrac{1}{x}$, $y = x$, $x = e$.

Rješenje. **1)** $x^2 + 4x = x + 4 \implies x(x+4) - (x+4) = 0 \implies (x-1)(x+4) = 0 \implies x_1 = -4, \quad x_2 = 1;\ y_1 = 0, \quad y_2 = 5.$

$P = -\int_{-4}^{0}(x^2 + 4x)dx + \int_{-4}^{1}(x+4)dx - \int_{0}^{1}(x^2+4x)dx = -\left(\frac{x^3}{3} + 2x^2\right)\Big|_{-4}^{0} + \left(\frac{x^2}{2} + 4x\right)\Big|_{-4}^{1} - \left(\frac{x^3}{3} + 2x^2\right)\Big|_{0}^{1} = -\left[0 - \left(-\frac{64}{3} + 32\right)\right] + \frac{1}{2} + 4 - 8 + 16 - \frac{1}{3} - 2 = \frac{31}{3} + 10 + \frac{1}{2} = \frac{61}{3} + \frac{1}{2} = \frac{122+3}{6} = \frac{125}{6}.$

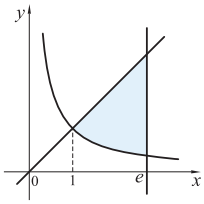


2) $P = \displaystyle\int_0^2 (e^x - e^{-x})dx = (e^x + e^{-x})\Big|_0^2 = e^2 + e^{-2} - e^0 - e^0 = e^2 + \frac{1}{e^2} - 2.$



3) $P = \displaystyle\int_0^1 \frac{dx}{1+x^2} = \operatorname{arc\,tg} x\Big|_0^1 = \frac{\pi}{4} - 0 = \frac{\pi}{4}.$

4) $P = \displaystyle\int_0^1 x dx + \int_1^e \frac{dx}{x} = \frac{x^2}{x}\Big|_0^1 + \ln|x|\Big|_1^e = \frac{1}{2} + \ln e - \ln 1 = \frac{1}{2} + 1 - 0 = \frac{3}{2}.$


y
0
1
e
x