

Zadatak 4.

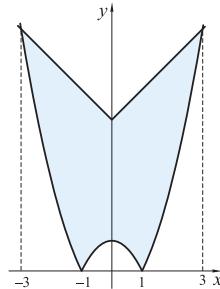
Izračunaj površinu lika omeđenog

- 1) krivuljama $y = |x^2 - 1|$ i $y = |x| + 5$;
- 2) grafovima funkcija $f(x) = 2 - |x|$,
 $g(x) = x^2$;
- 3) grafovima funkcija $f(x) = 1 + |x - 1|$,
 $g(x) = \frac{1}{2}x^2 - x$;
- 4) parabolama $y = x^2 - 2x + 2$ i
 $y = -x^2 + 4x + 2$;
- 5) krivuljom $y = x \cos x$ i pravcem $y = \frac{1}{2}x$ na intervalu $\langle -2, 2 \rangle$.

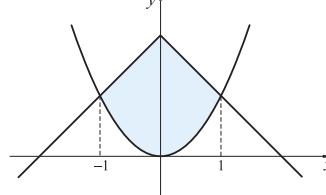
Rješenje.

1) $x^2 - 1 = x + 5 \implies x^2 - x - 6 = 0 \implies (x - 3)(x + 2) = 0 \implies x_1 = 3, x_2 = -2$.

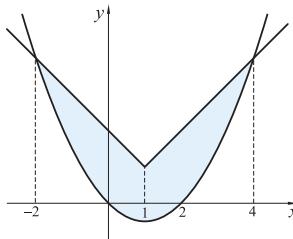
$$\begin{aligned} P &= 2P_1; \quad P_1 = \int_0^1 (x + 5 + x^2 - 1)dx + \int_1^3 (x + 5 + 1 - x^2)dx = \\ &= \int_0^1 (x^2 + x + 4)dx + \int_1^3 (6 + x - x^2)dx = \left(\frac{x^3}{3} + \frac{x^2}{2} + 4x \right) \Big|_0^1 + \left(6x + \frac{x^2}{2} - \frac{x^3}{3} \right) \Big|_1^3 = \\ &= \frac{1}{3} + \frac{1}{2} + 4 + 18 + \frac{9}{2} - \frac{27}{3} - 6 - \frac{1}{2} + \frac{1}{3} = 16 + \frac{9}{2} - \frac{25}{3} = 16 + 4\frac{1}{2} - 8\frac{1}{3} = \\ &= 12 + \frac{1}{2} - \frac{1}{3} = 12 + \frac{1}{6} = 12\frac{1}{6}. \quad P = 24\frac{1}{3}. \end{aligned}$$



2) $P = 2 \int_0^1 (2x - x - x^2)dx = 2 \left(2x - \frac{x^2}{2} - \frac{x^3}{3} \right) \Big|_0^1 = 2 \left(2 - \frac{1}{2} - \frac{1}{3} \right) = 2 \left(2 - \frac{5}{6} \right) = 2 \cdot \frac{7}{6} = \frac{14}{6} = \frac{7}{3}.$

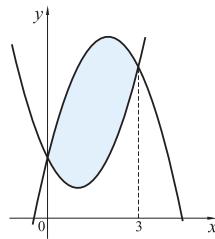


3) $P = 2 \left[\int_1^2 x dx + \int_2^4 \left(x - \frac{1}{2}x^2 + x \right) dx - \int_1^2 \left(\frac{1}{2}x^2 - x \right) dx \right] = 2 \left[\frac{x^2}{2} \Big|_1^2 + \left(x^2 - \frac{1}{6}x^3 \right) \Big|_2^4 - \left(\frac{1}{6}x^3 - \frac{x^2}{2} \right) \Big|_1^2 \right] = 2 \left(2 - \frac{1}{2} + 16 - \frac{1}{6} \cdot 64 - 4 + \frac{1}{6} \cdot 8 - \frac{8}{6} + 2 + \frac{1}{6} - \frac{1}{2} \right) = 2 \left(16 - 1 - \frac{21}{2} \right) = 2 \cdot \frac{9}{2} = 9.$



4) $x^2 - 2x + 2 = -x^2 + 4x + 2 \implies 2x^2 - 6x = 0 \implies 2x(x - 3) = 0 \implies x_1 = 0, x_2 = 3.$

$$P = \int_0^3 (-x^2 + 4x + 2 - x^2 + 2x - 2)dx = \int_0^3 (-2x^2 + 6x)dx = \left(-\frac{2}{3}x^3 + 3x^2 \right) \Big|_0^3 = -18 + 27 = 9.$$



5) $x \cos x = \frac{1}{2}x \implies x \left(\cos x - \frac{1}{2} \right) = 0 \implies x_1 = 0, x_2 = \frac{\pi}{3}.$

$$\begin{aligned} P &= 2 \int_0^{\pi/3} \left(x \cos x - \frac{1}{2}x \right) dx = 2 \int_0^{\pi/3} x \cos x dx - \int_0^{\pi/3} x dx \\ &= -\frac{x^2}{2} \Big|_0^{\pi/3} + 2 \int_0^{\pi/3} x \cos x dx = -\frac{\pi^2}{18} + 2 \int_0^{\pi/3} x \cos x dx \\ &= \left\{ \begin{array}{l} x = u \\ dx = dv \\ \cos x dx = dv \\ \sin x = v \end{array} \right\} = -\frac{\pi^2}{18} + 2x \sin x \Big|_0^{\pi/3} - 2 \int_0^{\pi/3} \sin x dx = \\ &= -\frac{\pi^2}{18} + \frac{2\pi}{3} \frac{\sqrt{3}}{2} + 2 \cos x \Big|_0^{\pi/3} = -\frac{\pi^2}{18} + \frac{\pi\sqrt{3}}{3} + 2 \left(\cos \frac{\pi}{3} - \cos 0 \right) = -\frac{\pi^2}{18} + \frac{\pi\sqrt{3}}{3} + 1 - 2 = \frac{\pi\sqrt{3}}{3} - \frac{\pi^2}{18} - 1 \approx 0.2655. \end{aligned}$$

