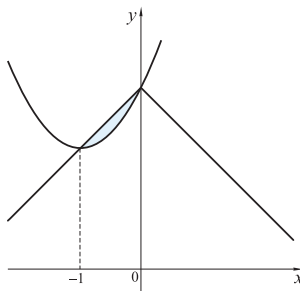


**Zadatak 5.** Izračunaj površinu lika omeđenog krivuljama:

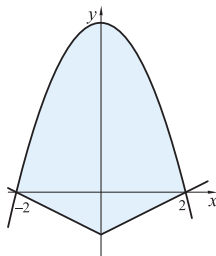
- 1)  $y = x^2 + 2x + 3$  i  $y = 3 - |x|$ ;
- 2)  $y = 4 - x^2$  i  $y = \frac{1}{2}|x| - 1$ ;
- 3)  $y = -x^2 + 6x - 5$ ,  $y = -x^2 + 4x - 3$  i  $y = 0$ ;
- 4)  $y = -x^2 + 6x - 5$ ,  $y = -x^2 + 4x - 3$  i  $y = 3x - 15$ ;
- 5)  $y = x^2 + 1$ ,  $y = x^2 + 2x$ ,  $x = 1$  i  $x = 2$ .

**Rješenje.**

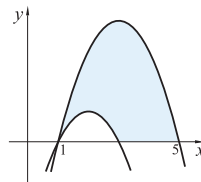
$$1) P = \int_{-1}^0 (x+3-x^2-2x-3)dx = \int_{-1}^0 (-x^2-x)dx = \left(-\frac{x^3}{3}-\frac{x^2}{2}\right)\Big|_{-1}^0 = -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}.$$



$$2) P = 2P_1; P_1 = \int_0^2 (4-x^2)dx - \int_0^2 \left(\frac{1}{2}x-1\right)dx = \left(4x-\frac{x^3}{3}-\frac{1}{4}x^2+x\right)\Big|_0^2 = 8 - \frac{8}{3} - 1 + 2 = \frac{19}{3} \implies P = \frac{38}{3}.$$

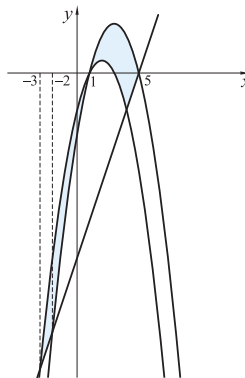


$$3) P = \int_1^5 (-x^2 + 6x - 5)dx - \int_1^3 (-x^2 + 4x - 3)dx = \left(-\frac{x^3}{3} + 3x^2 - 5x\right)\Big|_1^5 - \left(-\frac{x^3}{3} + 2x^2 - 3x\right)\Big|_1^3 = -\frac{125}{3} + 75 - 25 + \frac{1}{3} - 3 + 5 - \left(-9 + 18 - 9 + \frac{1}{3} - 2 + 3\right) = 52 - \frac{124}{3} - 1 - \frac{1}{3} = 51 - \frac{125}{3} = 51 - 41\frac{2}{3} = 10 - \frac{2}{3} = 9\frac{1}{3}.$$



$$\begin{aligned}
 4) \quad & -x^2 + 4x - 3 = 3x - 15 \implies x^2 - x - 12 = 0 \implies (x - 4)(x + 3) = 0 \\
 & \implies x_1 = -3, \quad x_2 = 4. \\
 & -x^2 + 6x - 5 = 3x - 15 \implies x^2 - 3x - 10 = 0 \implies (x - 5)(x + 2) = 0 \implies \\
 & x_1 = -2, \quad x_2 = 5.
 \end{aligned}$$

$$\begin{aligned}
 P_1 &= \int_{-3}^{-2} (-x^2 + 4x - 3 - 3x + 15) dx + \int_{-2}^1 (-x^2 + 4x - 3 + x^2 - 6x + 5) dx \\
 &= \int_{-3}^{-2} (-x^2 + x + 12) dx + \int_{-2}^1 (-2x + 2) dx = \left( -\frac{x^3}{3} + \frac{x^2}{2} + 12x \right) \Big|_{-3}^{-2} + (-x^2 + 2x) \Big|_{-2}^1 \\
 &= \left( \frac{8}{3} + 2 - 24 - \frac{9}{2} + 36 \right) + (-1 + 2 + 4 + 4) = \frac{19}{2} + \frac{8}{3} = \frac{73}{6} = 12\frac{1}{6} \\
 P_2 &= \int_1^5 (-x^2 + 6x - 5) dx - \int_1^3 (-x^2 + 4x - 3) dx - \int_3^4 (-x^2 + 4x - 3) dx - \int_4^5 (3x - 15) dx \\
 &= \frac{28}{3} - \left( -\frac{x^3}{3} + 2x^2 - 3x \right) \Big|_3^4 - \left( \frac{3}{2}x^2 - 15x \right) \Big|_4^5 \\
 &= \frac{28}{3} - \left( -\frac{64}{3} + 32 - 12 + 9 - 18 + 9 \right) - \left( \frac{75}{2} - 75 - 24 + 60 \right) = \frac{28}{3} + \frac{64}{3} - 20 + \frac{3}{2} \\
 &= \frac{92}{3} + \frac{3}{2} - 20 = 30\frac{2}{3} + 1\frac{1}{2} - 20 = 11 + \frac{2}{3} + \frac{1}{2} = 11 + \frac{7}{6} = 12\frac{1}{6} \\
 P &= P_1 + P_2 = 24\frac{1}{3}
 \end{aligned}$$



$$5) \quad P = \int_1^2 (x^2 + 2x - x^2 - 1) dx = \int_1^2 (2x - 1) dx = (x^2 - x) \Big|_1^2 = 4 - 2 - 1 + 1 = 2.$$

